

DYNAMIC MODELS FOR PANEL DATA: APPLICATION IN STATA

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1. Introduction

The aim of this note is to show some alternative estimates of a dynamic panel model: OLS, within, first-differences, IV, GMM. We use a dynamic panel model as an example of the bias deriving from endogeneity and of the most appropriate estimation methods.¹ Estimation methods like GMM can be used each time we tackle simultaneity or measurement errors, also in static model.

Moreover, we try to show how the same estimate may be obtained in different ways in Stata; this in order to better understand the syntax of two new commands, `ivreg2.ado` and `xtabond2.ado`, that represent a very useful extension of the Stata environment in dealing with endogeneity.

`ivreg2` (see C. F. Baum, M. E. Schaffer and S. Stillman (2003) Instrumental variables and GMM: Estimation and testing, *The Stata Journal*, 3, 1, 1-31) is a general estimating command. In fact, it implements a range of single-equation estimation methods for the linear regression model: OLS, instrumental variables (IV, also known as two-stage least squares, 2SLS), the two-step feasible generalized method of moments (GMM), limited-information maximum likelihood (LIML), and k-class estimators. It also implement a number of tests. Below some examples.

➤ The test of overidentifying restrictions. The joint null hypothesis is that the instruments are valid instruments, i.e., uncorrelated with the error term, and that the excluded instruments are correctly excluded from the estimated equation. Under the null, the test statistic is distributed as chi-squared in the number of overidentifying restrictions. A rejection casts doubt on the validity of the instruments. For the efficient GMM estimator, the test statistic is Hansen's J statistic², the minimised value of the GMM criterion function and saved in the output as `e(j)`, `e(jp)`, `e(jdf)` (see below). For the 2SLS estimator, the test statistic is Sargan's statistic³, typically calculated as NR² from a regression of the IV residuals on the full set of instruments and saved in the output as `e(sargan)`, `e(sarganp)`, `e(sargandf)`. The J statistic is consistent in the presence of heteroskedasticity and autocorrelation; Sargan's statistic is consistent if the disturbance is homoskedastic. Hence, with

¹ A good survey of these estimation methods applied to dynamic panel is Bond S.R. (2002), Dynamic panel data models: a guide to micro data methods and practice, WP Nuffield College, Oxford and Institute for Fiscal Studies.

² Hansen L. P. (1982) Large Sample Properties of Generalised Method of Moments Estimators, *Econometrica*, 50, 1029-54.

³ Sargan, J.D. (1958) The Estimation of Economic Relationships Using Instrumental Variables, *Econometrica*, 26, 393-415; (1998) Testing for misspecification after estimating using instrumental variables, in Maasoumi E. (ed.) Contributions to econometrics: John Denis Sargan, vol. 1, Cambridge University Press.

robust or cluster(.) options, Hansen's J statistic is reported; under the assumption of conditional homoskedasticity, Hansen's J statistic becomes Sargan's statistic⁴.

- Anderson canonical correlations likelihood-ratio test of whether the equation is identified, i.e., that the excluded instruments are relevant.⁵ This test assumes the regressors are distributed as multivariate normal and is saved in the output as `e(idstat)`, `e(idp)`, `e(iddf)`. The null hypothesis of the test is that the matrix of reduced form coefficients has rank=K-1, where K is the number of regressors, i.e, that the equation is underidentified. Under the null of underidentification, the statistic is distributed as chi-squared with degrees of freedom=(L-K+1), where L is the number of instruments (included+excluded). The statistic provides a measure of instrument relevance, and rejection of the null indicates that the model is identified. Note: a result of rejection of the null should be treated with caution, because weak instrument problems may still be present.
- Various identification statistics, if the option `ffirst` is used, saved in the matrix `e(first)`. For example: the Shea's (1997) "partial R-squared"⁶ that measures the instrument relevance taking into account intercorrelations among instruments; the more common form of "partial R-squared", given by the squared partial correlation between the excluded instruments and the endogenous regressor; the F-test of the excluded instruments that corresponds to the latter R-squared measure. The two measures of "partial R-squared" coincide when the model has only one endogenous regressor. If heteroskedastic-robust standard errors are used, the F-test is also heteroskedastic-robust.

`xtabond2` (see D. Roodman (2005) `xtabond2`: stata module to extend `xtabond` dynamic panel data estimator, Center for Global Development, Washington, DC, <http://econpapers.repec.org/software/bocbocode/s435901.htm>) offers the following opportunities.

- "Difference GMM" (GMM-dif) is the Arellano-Bond (1991)⁷ GMM estimator that treats the model as a system of equations, one for each time period; the equations differ only in their instrument/moment condition sets. The predetermined [correlated with past errors, but not with current and future errors: $E(\varepsilon_{it}|x_{is})=0 \forall t \geq s$ and $E(\varepsilon_{it}|x_{is}) \neq 0 \forall t < s, \forall i=1,\dots,N, \forall t, s=1,\dots,T]$ and endogenous [correlated with past and present errors, but not with current errors: $E(\varepsilon_{it}|x_{is})=0 \forall t > s$ and $E(\varepsilon_{it}|x_{is}) \neq 0 \forall t \leq s, \forall i=1,\dots,N, \forall t, s=1,\dots,T]$ variables in first differences are instrumented with suitable lags of their own levels. Strictly exogenous regressors [not correlated with errors in all temporal periods: $E(\varepsilon_{it}|x_{is})=0, \forall i=1,\dots,N, \forall t, s=1,\dots,T;$], as well as any other instruments, can enter the instrument matrix in the conventional instrumental variables fashion: in first differences, with one column per instrument.
- "System GMM" (GMM-sys) is the augmented version of GMM outlined in Arellano and Bover (1995) and fully developed in Blundell and Bond (1998) who more precisely articulated the necessary assumptions for this augmented estimator and tested it with Monte Carlo simulations.⁸ Lagged levels are often poor instruments for first differences, especially for variables that are close to a random walk. Thus, the original equations in levels can be added to the system, and the additional moment conditions could increase efficiency. In these equations, predetermined and endogenous variables in levels are instrumented with suitable lags of their own first differences.
- The updated version of `xtabond2` also allows for "Level GMM" (GMM-lev) in which IVs in first-differences or IVs in levels can be used. Pay attention that IVs in levels are not valid under the standard assumptions of dynamic panel data model.

⁴ See Hayashi F. (2000) *Econometrics*, Princeton University Press, pp. 227-228, 407, and 417.

⁵ Hall, A. R., G. D. Rudebusch, and D. W. Wilcox (1996) Judging instrument relevance in instrumental variables estimation, *International Economic Review*, 37, 2, 283-298.

⁶ Shea, J (1997) Instrument relevance in multivariate linear models: a simple measure, *Review of Economics and Statistics*, 49, 2, 348-352.

⁷ Arellano, M. and S. Bond (1991), Some tests of specification for panel data: Monte Carlo evidence and an application to employment equations, *The Review of Economic Studies*, 58, 277-97.

⁸ Arellano, M. and O. Bover (1995), Another look at the instrumental variable estimation of error-components models, *Journal of Econometrics*, 68, 29-51. Blundell, R. and S. Bond (1998), Initial conditions and moment restrictions in dynamic panel data models, *Journal of Econometrics*, 87, 115-43.

- Of course, many tests are included. The Arellano-Bond test for autocorrelation, saved in the output as `e(ari)`, `e(arip)`, `i=1, 2, ...`, and applied to the first-difference equation residuals: AR(1) is expected in first differences, while higher-order autocorrelation indicates that some lags of the dependent variable, which might be used as instruments, are in fact endogenous, thus bad instruments. The Sargan or Hansen J statistics, saved in the output as `e(sargan)`, `e(sarganp)`, `e(sar_df)`, to perform tests of over-identifying restrictions -whether the instruments, as a group, appear exogenous. For one-step non-robust estimation, the Sargan statistic is reported, which is the minimized value of the one-step GMM criterion function. For one-step robust estimation and for all two-step estimation, the Hansen J statistic is reported, which is the minimized value of the two-step GMM criterion function, and it is robust.

If compared to the original `xtabond` command available since stata 7, `xtabond2` presents many advantages.

- Also the GMM-sys and GMM-lev, and not only the GMM-dif, estimators are available.
- As GMM estimators, we have one- and two-step variants, with two-step estimates asymptotically more efficient⁹. However, the dependence of the two-step weight matrix on estimated parameters makes asymptotic distribution approximations less reliable in small samples: in the `xtabond` command, the two-step estimates of the standard errors tend to be severely downward biased, or the t-ratios tend to be upward biased. In `xtabond2`, the finite-sample correction for the asymptotic variance of the two-step estimator suggested by Windmeijer (2005)¹⁰ is available.
- There is full control over the instrument matrix, thanks to a syntax that almost completely decouples specification of regressors from specification of instruments (the cost is a loss of parsimony, because most variables used will appear twice).
- The constant term is not included in the first-difference equations in GMM-dif, since this would be equivalent to including time (a trend) as a variable in the levels model¹¹. Rather, the constant is differenced out. By default, the constant term is included as a regressor and IV-style instrument in the levels equation, but it can be suppressed with the `noconstant` option.
- It is written to take advantage of the Mata (matrix) programming language first included in Stata 9.0. It works in Stata 7 and 8, but runs much faster in Stata 9 or later. The `xtabond2 nomata` option can be used to prevent the use of Mata even when it is available (this is convenient in large sample because it could exist a difficult trade-off between speed and memory).

2. The data-set

The data-set `abdata.dta` contains the data used in Arellano-Bond (1991) and Blundell-Bond (1998). The aim is to estimate a model for employment in a panel of companies in UK:

id and *year* are the individual and temporal indicators;

n is the log of employees (*emp*);

k is the log of gross capital stock (*cap*);

ys is the log of output of each industry (*indoutpt*, included in order to capture demand shocks);

w is the log of per-employee real wage (*wage*, deflated by output prices).

Originally, the data have been employed by Layard R.-Nickell S.J. (1986) “Unemployment in Britain, Econometrica”, 53, Supplement, 5121-69.

```
use abdata, clear
```

⁹ Many applied works use one-step estimates because simulation studies suggest very modest efficiency gains from two-step, even in presence of heteroskedasticity. See, for example, Blundell R.W., Bond S.R. and Windmeijer F. (2000), Estimation in dynamic panel data models: improving on the performance of the standard GMM estimator, in B. Baltagi (ed) *Advances in Econometrics, Volume 15: Non Stationary panels, panel cointegration, and dynamic panels*, JAI Elsevier science.

¹⁰ Windmeijer, F. 2005, A Finite Sample correction for the variance of linear efficient two-step GMM estimators, *Journal of Econometrics*, 126, 25-51.

¹¹ This is not possible in `xtabond` and in DPD, the original software that implemented dynamic panel estimators (Doornik, J.A., M. Arellano, and S. Bond (2002) Panel data estimation using DPD for Ox, <http://www.nuff.ox.ac.uk/Users/Doornik>).

```

descr
Contains data from C:\banchedati\GMM\abdata.dta
  obs:           1,031
  vars:            21
  size:        93,821 (99.1% of memory free)
-----
variable name   storage  display      value
variable type    format     label      variable label
-----
c1             str9    %9s
year            float    %9.0g
rec              float    %9.0g
yearm1          float    %9.0g
id               float    %9.0g
nL1              float    %9.0g
nL2              float    %9.0g
wL1              float    %9.0g
kL1              float    %9.0g
kL2              float    %9.0g
ysL1             float    %9.0g
ysL2             float    %9.0g
ind              int      %8.0g      industry
emp              float    %9.0g      employment
wage             float    %9.0g      real wage
cap              float    %9.0g      gross capital stock
indoutpt         float    %9.0g      industry output
n                float    %9.0g      log(employment)
w                float    %9.0g      log(real wage)
k                float    %9.0g      log(gross capital stock)
ys              float    %9.0g      log(industry output)
-----
Sorted by: id year

```

Variables like *nL1* are just lags, generated, for example, as:

```

g nL1=l1.n
(140 missing values generated)
g nL2=l2.n
(280 missing values generated)

```

This in case some commands do not understand time-series operator.

The data-set is unbalanced.

```

. tsset id year
  panel variable: id, 1 to 140
  time variable: year, 1976 to 1984

. xtdes, pattern(1000)

  id: 1, 2, ..., 140                                n =
  year: 1976, 1977, ..., 1984                          T =
  Delta(year) = 1; (1984-1976)+1 = 9
  (id*year uniquely identifies each observation)

Distribution of T_i: min      5%      25%      50%      75%      95%      max
                           7       7       7       7       8       9       9

  Freq.  Percent   Cum. | Pattern
-----+-----
  62     44.29  44.29 | 1111111..
  39     27.86  72.14 | .1111111.
  19     13.57  85.71 | .11111111
  14     10.00  95.71 | 111111111
    4      2.86  98.57 | 111111111.
    2      1.43 100.00 | ..11111111
-----+-----
  140    100.00          | XXXXXXXXXX

```

3. Model estimates

The model presented in Arellano-Bond (1991) is:

$$n_{it} = \alpha_1 n_{it-1} + \alpha_2 n_{it-2} + \beta_1 w_{it} + \beta_2 w_{it-1} + \beta_3 k_{it} + \beta_4 k_{it-1} + \beta_5 k_{it-2} + \beta_6 y_{it} + \beta_7 y_{it-1} + \beta_8 y_{it-2} + \lambda_t + \eta_i + \nu_{it}$$

The model estimated in Blundell-Bond (1998) is:

$$n_{it} = a_1 n_{it-1} + b_1 w_{it} + b_2 w_{it-1} + b_3 k_{it} + b_4 k_{it-1} + \lambda_t + \eta_i + \nu_{it}$$

The procedure `dtime.ado` allows for generating the temporal dummies used to estimate λ_t , a time effect that is common to all the companies.

```
. dtime year 1976 1984
```

The following options influence the behaviour of Stata in running complex models and the behaviour of the Mata code.

`set matsize` sets the maximum number of variables that can be included in any of Stata's estimation commands. For Stata/SE the default value is 400, but it may be changed upward or downward, with upper limit equal to 11,000.

`mata query` shows the values of Mata's system parameters.

`mata set` sets the value of the system parameters. For example, `mata set matocache` specifies the maximum amount of memory, in kilobytes, that may be consumed before Mata starts looking to drop autoloaded functions that are not currently being used. The default value is 400, meaning 400 kilobytes. This parameter affects the efficiency with which Stata runs. Larger values cannot hurt, but once `matocache` is large enough, larger values will not improve performance. As another example, `mata set matafavor` specifies whether, when executing code, Mata should favor conserving memory (`space`) or running quickly (`speed`). The default setting is `space`. Switching to `speed` will make Mata, in a few instances, run a little quicker but consume more memory.

```
. *set matsize 7000
. mata: mata query
. *mata: mata set matocache 900
. mata: mata set matafavor speed
. *mata: mata set matafavor space
```

3.1. OLS estimates: Arellano-Bond (1991), Table 5, col. (g), p. 292

```
. reg n l(1/2).n l(0/1).w l(0/2).(k ys) tau1979-tau1984, cluster(id)
```

Linear regression	Number of obs = 751
	F(16, 139) = 13990.88
	Prob > F = 0.0000
	R-squared = 0.9944
	Root MSE = .10158

Number of clusters (id) = 140

	Robust					
n	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
L1.	1.044643	.0517969	20.17	0.000	.9422313	1.147055
L2.	-.0765426	.0488082	-1.57	0.119	-.1730451	.0199598
w						
--.	-.5236727	.1740911	-3.01	0.003	-.8678817	-.1794637
L1.	.4767538	.1717904	2.78	0.006	.1370937	.8164139
k						
--.	.3433951	.048649	7.06	0.000	.2472074	.4395829
L1.	-.2018991	.0650327	-3.10	0.002	-.3304803	-.073318

L2.	-.1156467	.0358966	-3.22	0.002	-.1866206	-.0446727
ys						
--.	.4328752	.17894	2.42	0.017	.079079	.7866715
L1.	-.7679125	.2514336	-3.05	0.003	-1.265041	-.2707836
L2.	.3124721	.1322678	2.36	0.020	.0509551	.5739891
tau1979	.0158888	.0090408	1.76	0.081	-.0019865	.0337641
tau1980	.0219933	.0149899	1.47	0.145	-.0076444	.0516309
tau1981	-.0221532	.0242324	-0.91	0.362	-.0700648	.0257585
tau1982	-.0150344	.0214242	-0.70	0.484	-.0573938	.0273251
tau1983	.0073931	.01963	0.38	0.707	-.0314189	.0462052
tau1984	.0153956	.0204269	0.75	0.452	-.024992	.0557832
_cons	.2747256	.3194854	0.86	0.391	-.3569538	.906405

. abar, lags(2)

Arellano-Bond test for AR(1): z = 1.29 Pr > z = 0.1978

Arellano-Bond test for AR(2): z = -1.03 Pr > z = 0.3039

. ivreg2 n l(1/2).n l(0/1).w l(0/2).(k ys) tau1979-tau1984, cluster(id) small

OLS regression with robust standard errors

Number of clusters (id) = 140

Number of obs = 751

F(16, 139) = 13990.88

Prob > F = 0.0000

Total (centered) SS = 1350.891752
 Total (uncentered) SS = 2122.555626
 Residual SS = 7.573781642

Centered R2 = 0.9944

Uncentered R2 = 0.9964

Root MSE = .1016

	Robust					
n	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
n						
L1.	1.044643	.0517969	20.17	0.000	.9422313	1.147055
L2.	-.0765426	.0488082	-1.57	0.119	-.1730451	.0199598
w						
--.	-.5236727	.1740911	-3.01	0.003	-.8678817	-.1794637
L1.	.4767538	.1717904	2.78	0.006	.1370937	.8164139
k						
--.	.3433951	.048649	7.06	0.000	.2472074	.4395829
L1.	-.2018991	.0650327	-3.10	0.002	-.3304803	-.073318
L2.	-.1156467	.0358966	-3.22	0.002	-.1866206	-.0446727
ys						
--.	.4328752	.17894	2.42	0.017	.079079	.7866715
L1.	-.7679125	.2514336	-3.05	0.003	-1.265041	-.2707836
L2.	.3124721	.1322678	2.36	0.020	.0509551	.5739892
tau1979	.0158888	.0090408	1.76	0.081	-.0019865	.0337641
tau1980	.0219933	.0149899	1.47	0.145	-.0076444	.0516309
tau1981	-.0221532	.0242324	-0.91	0.362	-.0700648	.0257585
tau1982	-.0150344	.0214242	-0.70	0.484	-.0573938	.0273251
tau1983	.0073931	.01963	0.38	0.707	-.0314189	.0462052
tau1984	.0153956	.0204269	0.75	0.452	-.024992	.0557832
_cons	.2747256	.3194854	0.86	0.391	-.3569538	.906405

. abar, lags(2)

Arellano-Bond test for AR(1): z = 1.29 Pr > z = 0.1978

Arellano-Bond test for AR(2): z = -1.03 Pr > z = 0.3039

. ivreg2 n l(1/2).n l(0/1).w l(0/2).(k ys) tau1979-tau1984, cluster(id)

OLS regression with robust standard errors

Number of clusters (id) = 140

Number of obs = 751

F(16, 139) = 13990.88

Total (centered) SS	=	1350.891752	Prob > F	=	0.0000
Total (uncentered) SS	=	2122.555626	Centered R2	=	0.9944
Residual SS	=	7.573781642	Uncentered R2	=	0.9964
			Root MSE	=	.1004

n	Robust					
	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
n						
L1.	1.044643	.0510581	20.46	0.000	.9445709	1.144715
L2.	-.0765426	.048112	-1.59	0.112	-.1708404	.0177551
w						
--.	-.5236727	.1716079	-3.05	0.002	-.860018	-.1873274
L1.	.4767538	.16934	2.82	0.005	.1448535	.8086542
k						
--.	.3433951	.0479551	7.16	0.000	.2494048	.4373854
L1.	-.2018991	.0641051	-3.15	0.002	-.3275428	-.0762555
L2.	-.1156467	.0353846	-3.27	0.001	-.1849992	-.0462942
ys						
--.	.4328752	.1763877	2.45	0.014	.0871617	.7785888
L1.	-.7679125	.2478472	-3.10	0.002	-1.253684	-.2821409
L2.	.3124721	.1303812	2.40	0.017	.0569297	.5680146
tau1979	.0158888	.0089119	1.78	0.075	-.0015781	.0333557
tau1980	.0219933	.0147761	1.49	0.137	-.0069673	.0509538
tau1981	-.0221532	.0238867	-0.93	0.354	-.0689703	.0246639
tau1982	-.0150344	.0211186	-0.71	0.477	-.0564261	.0263574
tau1983	.0073931	.01935	0.38	0.702	-.0305322	.0453185
tau1984	.0153956	.0201355	0.76	0.445	-.0240693	.0548605
_cons	.2747256	.3149284	0.87	0.383	-.3425227	.8919739

. abar, lags(2)

Arellano-Bond test for AR(1): z = 1.47 Pr > z = 0.1421
 Arellano-Bond test for AR(2): z = -1.03 Pr > z = 0.3036

.
 . xtabond2 n l(1/2).n l(0/1).w l(0/2).(k ys) tau1979-tau1984, iv(l(1/2).n l(0/1).w l(0/2).(k ys) tau1979-tau1984, eq(level)) arle robust
 Favoring speed over space. To switch, type or click on mata: mata set matafavor space.

Arellano-Bond dynamic panel-data estimation, one-step system GMM results

Group variable: id	Number of obs	=	751
Time variable : year	Number of groups	=	140
Number of instruments = 17	Obs per group: min	=	5
Wald chi2(16) = 230379.33	avg	=	5.36
Prob > chi2 = 0.000	max	=	7

n	Robust					
	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
n						
L1.	1.044643	.0510581	20.46	0.000	.9445709	1.144715
L2.	-.0765426	.048112	-1.59	0.112	-.1708404	.0177551
w						
--.	-.5236727	.1716079	-3.05	0.002	-.860018	-.1873274
L1.	.4767538	.16934	2.82	0.005	.1448535	.8086542
k						
--.	.3433951	.0479551	7.16	0.000	.2494048	.4373854
L1.	-.2018991	.0641051	-3.15	0.002	-.3275428	-.0762555
L2.	-.1156467	.0353846	-3.27	0.001	-.1849992	-.0462942
ys						
--.	.4328754	.1763877	2.45	0.014	.0871618	.778589
L1.	-.7679126	.2478473	-3.10	0.002	-1.253684	-.2821409
L2.	.3124721	.1303812	2.40	0.017	.0569297	.5680146
tau1979	.0158888	.0089119	1.78	0.075	-.0015781	.0333557
tau1980	.0219933	.0147761	1.49	0.137	-.0069673	.0509538
tau1981	-.0221532	.0238867	-0.93	0.354	-.0689703	.0246639
tau1982	-.0150344	.0211186	-0.71	0.477	-.0564261	.0263574

```

tau1983 |   .0073931    .01935    0.38    0.702   -.0305322    .0453185
tau1984 |   .0153956    .0201355    0.76    0.445   -.0240693    .0548605
_cons |   .2747254    .3149284    0.87    0.383   -.3425228    .8919737
-----
Hansen test of overid. restrictions: chi2(0) =   0.00      Prob > chi2 =   .
Arellano-Bond test for AR(1) in levels:           z =     1.47  Pr > z =  0.142
Arellano-Bond test for AR(2) in levels:           z =   -1.03  Pr > z =  0.304
-----
. predict yhat
(280 missing values generated)

. qui corr yhat n if e(sample)

. scalar XTAB_r2=r(rho)^2

. drop yhat

. di XTAB_r2
.99439349

```

Considerations.

- Years 1976 and 1977 are lost due to the use of 2 lags as explanatory variables. the dummy for 1978 is not included because we leave the constant in the model. The inclusion of right dummies (1979-1984, especially 1979) is fundamental in order to obtain the same results as Arellano-Bond paper.
- By default, both `ivreg2` and `xtabond2` use asymptotic standard errors, the same reported by Arellano-Bond. In order to reproduce the same standard errors as `reg`, we need the `small` option. This allows for coincidence between `reg` and `ivreg2` also when the options `robust` or `cluster(.)` are used¹². On the contrary, `xtabond2`, `robust` ignores `small`. NOTE: `xtabond2` does not have option `cluster(.)`: `robust` in `xtabond2` is equal to `cluster(.)` in `ivreg2`.¹³ Cluster is the option used by Arellano-Bond.
- The test for second-order (or higher order) serial correlation of the residuals is automatically done by `xtabond2`; option `arlevels` specifies that the autocorrelation tests should be applied to the residuals from the levels, not first-difference, equations. In the `reg` and `ivreg2` cases, the procedure `abar.ado`, written by D. Roodman, Center for Global Development, Washington, DC, is tyo be used. This procedure, originally proposed for a particular linear GMM dynamic panel data estimator, can be applied to linear GMM regressions in general, and thus to OLS and 2SLS regressions, which can be seen as special cases of linear GMM. It is appropriate for both time-series and cross-section time-series (panel) regressions. It is consistent in the presence of various patterns of error covariance when the options `robust` or `cluster` are used.
- In `xtabond2` the option `ivstyle()` specifies a set of variables to serve as standard instruments, with one column in the instrument matrix per variable. Normally, strictly exogenous regressors are included in `ivstyle` options in order to enter the instrument matrix, as well as being listed before the main comma of the command line. The suboption `eq(.)` specifies which set of equation(s) should use the instruments: first-difference only (`equation(diff)`), levels only

¹² In `ivreg2` and `xtabond2` the MSE is, by default, defined as the squared root of RSS/NT; if `small` is specified, it is the squared root of RSS/(NT-K). With `robust` and `cluster` there is, by default, no degrees of freedom adjustment. If `small` `robust` is specified, the finite sample adjustement to the robust variance-covariance matrix is NT/(NT-K). If `small` `cluster` is specified, the finite sample adjustement to the robust variance-covariance matrix is (NT-1)/(NT-K)× N_c/(N_c-1), where N_c is the total number of clusters.

¹³ Summarising, we have the following combinations:

```

reg = ivreg2, small = xtabond2, small
reg, robust = ivreg2, small robust and reg, cluster = ivreg2, small cluster
ivreg2, robust
ivreg2, cluster = xtabond2, robust

```

(equation(level)), or both (equation(both), the default). By default, the instruments are first-differenced for use in the first-difference equations and taken as is for instrumenting the levels equations. The eq(.) option can be useful when x is predetermined: since it is assumed to be uncorrelated with the contemporaneous and future error term, it is a valid instrument for the levels equations; however, x becomes endogenous in first differences, so Δx is not a valid instrument for the first-difference equations.

- In xtabond2 the R-squared is not available. Thus, we compute it as the squared correlation coefficient between actual and fitted values.

3.2. Within estimates: Arellano-Bond (1991), Table 5, col. (h), p. 292

```
. xtreg n l(1/2).n l(0/1).w l(0/2).(k ys) tau1979-tau1984, fe cluster(id)

Fixed-effects (within) regression
Group variable (i): id
Number of obs = 751
Number of groups = 140

R-sq: within = 0.7973
      between = 0.9809
      overall = 0.9758
Obs per group: min = 5
                  avg = 5.4
                  max = 7
F(16,139) = 152.18
Prob > F = 0.0000
corr(u_i, Xb) = 0.5459
(Std. Err. adjusted for 140 clusters in id)
-----+
          | Robust
       n | Coef. Std. Err.      t   P>|t| [95% Conf. Interval]
-----+
       n |
L1. | .7329476  .0596831    12.28  0.000    .6149436  .8509516
L2. | -.1394773  .0781564   -1.78  0.077   -.2940065  .0150519
       w |
--. | -.5597445  .1596195   -3.51  0.001   -.8753406  -.2441484
L1. | .3149987  .1430587    2.20  0.029    .0321463  .5978511
       k |
--. | .3884188  .056928    6.82  0.000    .275862  .5009756
L1. | -.0805185  .0538774   -1.49  0.137   -.1870436  .0260066
L2. | -.0278013  .0426222   -0.65  0.515   -.1120728  .0564703
       ys |
--. | .468666  .1712492    2.74  0.007    .1300759  .8072561
L1. | -.6285587  .2066106   -3.04  0.003   -.1.037065  -.2200527
L2. | .0579764  .1326758    0.44  0.663   -.2043473  .3203001
tau1979 | .0046562  .0092572    0.50  0.616   -.0136469  .0229593
tau1980 | .0112327  .0158528    0.71  0.480   -.0201111  .0425765
tau1981 | -.0253693  .0249902   -1.02  0.312   -.0747794  .0240408
tau1982 | -.0343973  .022882   -1.50  0.135   -.0796391  .0108444
tau1983 | -.0280344  .0262074   -1.07  0.287   -.079851  .0237822
tau1984 | -.0119152  .0281652   -0.42  0.673   -.0676028  .0437723
       _cons | 1.79212  .633652    2.83  0.005    .5392775  3.044963
-----+
sigma_u | .22568151
sigma_e | .09395847
rho | .85227336 (fraction of variance due to u_i)
-----+
*. *qui within id n nL1 nL2 w wL1 k kL1 kL2 ys ysL1 ysL2 tau1979 tau1980 tau1981 tau1982
tau1983 tau1984 if nL2!=.
*. qui within id n nL1 nL2 w wL1 k kL1 kL2 ys ysL1 ysL2 tau1979 tau1980 tau1981 tau1982 tau1983
tau1984 if e(sample)
*
*. ivreg2 n_WD nL1_WD nL2_WD w_WD wL1_WD k_WD kL1_WD kL2_WD ys_WD ysL1_WD ysL2_WD tau1979_WD-
tau1984_WD, small noconstant cluster(id)
OLS regression with robust standard errors
-----+
```

Number of clusters (id) = 140	Number of obs = 751
	F(16, 139) = 152.38
	Prob > F = 0.0000
Total (centered) SS = 25.91406328	Centered R2 = 0.7973
Total (uncentered) SS = 25.91406328	Uncentered R2 = 0.7973
Residual SS = 5.252775373	Root MSE = .08454

n_WD	Coef.	Robust Std. Err.	t	P> t	[95% Conf. Interval]
nL1_WD	.7329477	.0596424	12.29	0.000	.615024 .8508714
nL2_WD	-.1394773	.0781033	-1.79	0.076	-.2939013 .0149468
w_WD	-.5597445	.1595108	-3.51	0.001	-.8751258 -.2443632
wL1_WD	.3149987	.1429613	2.20	0.029	.0323388 .5976586
k_WD	.3884188	.0568893	6.83	0.000	.2759386 .500899
kL1_WD	-.0805185	.0538407	-1.50	0.137	-.1869711 .0259341
kL2_WD	-.0278013	.0425932	-0.65	0.515	-.1120155 .056413
ys_WD	.468666	.1711327	2.74	0.007	.1303063 .8070257
ysL1_WD	-.6285587	.20647	-3.04	0.003	-1.036787 -.2203307
ysL2_WD	.0579764	.1325855	0.44	0.663	-.2041688 .3201216
tau1979_WD	.0046562	.0092509	0.50	0.616	-.0136344 .0229468
tau1980_WD	.0112327	.015842	0.71	0.479	-.0200898 .0425552
tau1981_WD	-.0253693	.0249732	-1.02	0.311	-.0747458 .0240072
tau1982_WD	-.0343973	.0228664	-1.50	0.135	-.0796083 .0108136
tau1983_WD	-.0280344	.0261895	-1.07	0.286	-.0798157 .0237469
tau1984_WD	-.0119152	.028146	-0.42	0.673	-.0675649 .0437344

```
.
. ivreg2 n_WD nL1_WD nL2_WD w_WD wL1_WD k_WD kL1_WD kL2_WD ys_WD ysL1_WD ysL2_WD tau1979_WD-
tau1984_WD, noconstant cluster(id)
```

OLS regression with robust standard errors

Number of clusters (id) = 140	Number of obs = 751
	F(16, 139) = 152.38
	Prob > F = 0.0000
Total (centered) SS = 25.91406328	Centered R2 = 0.7973
Total (uncentered) SS = 25.91406328	Uncentered R2 = 0.7973
Residual SS = 5.252775373	Root MSE = .08363

n_WD	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]
nL1_WD	.7329477	.0588318	12.46	0.000	.6176395 .8482558
nL2_WD	-.1394773	.0770417	-1.81	0.070	-.2904761 .0115216
w_WD	-.5597445	.1573427	-3.56	0.000	-.8681306 -.2513584
wL1_WD	.3149987	.1410181	2.23	0.025	.0386083 .5913891
k_WD	.3884188	.056116	6.92	0.000	.2784334 .4984042
kL1_WD	-.0805185	.0531089	-1.52	0.129	-.1846099 .023573
kL2_WD	-.0278013	.0420142	-0.66	0.508	-.1101476 .0545451
ys_WD	.468666	.1688066	2.78	0.005	.1378112 .7995208
ysL1_WD	-.6285587	.2036636	-3.09	0.002	-1.027732 -.2293853
ysL2_WD	.0579764	.1307834	0.44	0.658	-.1983543 .3143072
tau1979_WD	.0046562	.0091251	0.51	0.610	-.0132287 .0225411
tau1980_WD	.0112327	.0156267	0.72	0.472	-.0193951 .0418605
tau1981_WD	-.0253693	.0246338	-1.03	0.303	-.0736506 .022912
tau1982_WD	-.0343973	.0225556	-1.53	0.127	-.0786055 .0098108
tau1983_WD	-.0280344	.0258335	-1.09	0.278	-.0786672 .0225984
tau1984_WD	-.0119152	.0277634	-0.43	0.668	-.0663305 .0425001

```
.
. xtabond2 n_WD nL1_WD nL2_WD w_WD wL1_WD k_WD kL1_WD kL2_WD ys_WD ysL1_WD ysL2_WD tau1979_WD-
tau1984_WD, iv(nL1_WD nL2_WD w_WD wL1_WD k_WD kL1_WD kL2_WD ys_WD ysL1_WD ysL2_WD tau1979_WD-
tau1984_WD, eq(level)) noconstant arle robust
```

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Arellano-Bond dynamic panel-data estimation, one-step system GMM results

Group variable: id		Number of obs	=	751
Time variable : year		Number of groups	=	140
Number of instruments = 16		Obs per group: min =		5
Wald chi2(16) = 2505.81		avg =		5.36
Prob > chi2 = 0.000		max =		7

		Robust				
	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
nL1_WD	.7329477	.0588318	12.46	0.000	.6176395	.8482558
nL2_WD	-.1394773	.0770417	-1.81	0.070	-.2904761	.0115216
w_WD	-.5597445	.1573427	-3.56	0.000	-.8681306	-.2513584
wL1_WD	.3149987	.1410181	2.23	0.025	.0386083	.5913891
k_WD	.3884188	.056116	6.92	0.000	.2784334	.4984042
kL1_WD	-.0805185	.0531089	-1.52	0.129	-.1846099	.023573
kL2_WD	-.0278013	.0420142	-0.66	0.508	-.1101476	.0545451
ys_WD	.468666	.1688066	2.78	0.005	.1378112	.7995208
ysL1_WD	-.6285587	.2036636	-3.09	0.002	-1.027732	-.2293853
ysL2_WD	.0579764	.1307834	0.44	0.658	-.1983543	.3143072
tau1979_WD	.0046562	.0091251	0.51	0.610	-.0132287	.0225411
tau1980_WD	.0112327	.0156267	0.72	0.472	-.0193951	.0418605
tau1981_WD	-.0253693	.0246338	-1.03	0.303	-.0736506	.022912
tau1982_WD	-.0343973	.0225556	-1.53	0.127	-.0786055	.0098108
tau1983_WD	-.0280344	.0258335	-1.09	0.278	-.0786672	.0225984
tau1984_WD	-.0119152	.0277634	-0.43	0.668	-.0663305	.0425001

Hansen test of overid. restrictions: chi2(0) = 0.00 Prob > chi2 = .

Arellano-Bond test for AR(1) in levels: z = -3.87 Pr > z = 0.000
 Arellano-Bond test for AR(2) in levels: z = -4.68 Pr > z = 0.000

```
. predict yhat
(280 missing values generated)

. qui corr yhat n_WD if e(sample)

. scalar XTAB_r2=r(rho)^2

. drop yhat

. di XTAB_r2
.7973002
```

Considerations.

- In Stata 9 `xtreg` allows for robust/cluster standard errors and it understands time-series operators.
- `ivreg2` and `xtabond2` require within data transformation (`within.ado` procedure). Note that the “right” sample must be used, either by imposing a condition on the data availability or by imposing to use the same estimation sample of `xtreg`. Note also that all the variables, temporal dummies included, must be transformed. When using `cluster`, is the downwards bias in the standard errors relevant as seen in `lecture_panel_static_application`?¹⁴ Hint: try to run the same estimation commands with the option `robust`.

¹⁴ With `small` option, the MSE is defined as the squared root of $\text{RSS}/(\text{NT}-\text{K})$. If `cluster` is specified, the MSE is obtained by a finite sample adjustment to the variance-covariance matrix: $\text{RSS}/\text{NT} \times (\text{NT}-1)/(\text{NT}-\text{K}) \times N_c/(N_c-1)$, where N_c is the total number of clusters. In this case, we need a correction different from the ones used in `lecture_panel_static_application`. In `ivreg2` without the option `small` and in `xtabond2` the standard errors are asymptotic; thus, the MSE is defined as the squared root of RSS/NT , both without and with `cluster` option, because `cluster` does not creates any degrees of freedom adjustment.

- The `abar` procedure does not work in this case, and the output of autocorrelation test reported by `xtabond2` must to be not considered. This test is not appropriate for fixed-effects regressions for dynamic models, assuming those are done via a mean-deviation transformation. This is because the Arellano-Bond AR(p) test assumes that right-hand-side variables are not “post-determined” i.e. not correlated with future errors. In a dynamic setting, future values of regressors can depend on future errors. And after the mean-deviations transformation, future values of the original regressors affect current values of the transformed versions.

3.3. First-differences estimates

```
. xtivreg n (l(1/2).n l(0/1).w l(0/2).(k ys) tau1979-tau1984 = l(1/2).n l(0/1).w l(0/2).(k ys)
tau1979-tau1984), fd noconst
```

First-differenced IV regression		Number of obs		= 611			
Group variable: id		Number of groups		= 140			
R-sq:			Obs per group:				
within = 0.1603			min = 4				
between = 0.5014			avg = 4.4				
overall = 0.8637			max = 6				
			chi2(16) = 699.56				
corr(u_i, Xb) = 0.7056			Prob > chi2 = 0.0000				
d.n	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]		
n							
LD.	.1218396	.0434764	2.80	0.005	.0366275 .2070517		
L2D.	-.0556518	.0419881	-1.33	0.185	-.137947 .0266434		
w							
D1.	-.5551606	.055246	-10.05	0.000	-.6634407 -.4468804		
LD.	.1238423	.0637157	1.94	0.052	-.0010381 .2487227		
k							
D1.	.3938555	.0307857	12.79	0.000	.3335167 .4541943		
LD.	.1252152	.0328176	3.82	0.000	.0608938 .1895365		
L2D.	.040182	.0329258	1.22	0.222	-.0243515 .1047154		
ys							
D1.	.5692903	.117116	4.86	0.000	.3397473 .7988334		
LD.	-.3754761	.1282028	-2.93	0.003	-.6267491 -.1242032		
L2D.	.0272238	.1313197	0.21	0.836	-.230158 .2846056		
tau1979							
D1.	-.0000954	.0125153	-0.01	0.994	-.024625 .0244342		
tau1980							
D1.	.0068434	.0181881	0.38	0.707	-.0288047 .0424914		
tau1981							
D1.	-.0229064	.0265084	-0.86	0.388	-.0748619 .0290491		
tau1982							
D1.	-.0495964	.0317921	-1.56	0.119	-.1119078 .012715		
tau1983							
D1.	-.0591959	.0373131	-1.59	0.113	-.1323283 .0139366		
tau1984							
D1.	-.0529525	.0419136	-1.26	0.206	-.1351017 .0291967		
<hr/>							
sigma_u	.71755048						
sigma_e	.10926003						
rho	.97733982	(fraction of variance due to u_i)					
<hr/>							

Instrumented: L.n L2.n w L.w k L.k L2.k ys L.ys L2.ys tau1979 tau1980 tau1981 tau1982

tau1983 tau1984

Instruments: L.n L2.n w L.w k L.k L2.k ys L.ys L2.ys tau1979 tau1980 tau1981 tau1982

tau1983 tau1984

```
. ivreg2 d.n dl.n dl2.n d.w dl.w d.k dl.k dl2.k d.ys dl.ys dl2.ys d.(tau1979 tau1980 tau1981
tau1982 tau1983 tau1984), noconst small
```

Ordinary Least Squares (OLS) regression

Total (centered) SS	=	12.59997848	Number of obs	=	611
Total (uncentered) SS	=	15.45414725	F(16, 595)	=	43.72
Residual SS	=	7.102963775	Prob > F	=	0.0000
			Centered R2	=	0.4363
			Uncentered R2	=	0.5404
			Root MSE	=	.1093

D.n	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
n					
LD.	.1218396	.0434764	2.80	0.005	.0364538 .2072254
L2D.	-.0556518	.0419881	-1.33	0.186	-.1381148 .0268111
w					
D1.	-.5551606	.055246	-10.05	0.000	-.6636614 -.4466597
LD.	.1238423	.0637157	1.94	0.052	-.0012926 .2489773
k					
D1.	.3938555	.0307857	12.79	0.000	.3333937 .4543173
LD.	.1252152	.0328176	3.82	0.000	.0607627 .1896676
L2D.	.040182	.0329258	1.22	0.223	-.024483 .104847
ys					
D1.	.5692903	.117116	4.86	0.000	.3392794 .7993013
LD.	-.3754761	.1282028	-2.93	0.004	-.6272613 -.123691
L2D.	.0272238	.1313197	0.21	0.836	-.2306826 .2851302
tau1979					
D1.	-.00000954	.0125153	-0.01	0.994	-.024675 .0244842
tau1980					
D1.	.0068434	.0181881	0.38	0.707	-.0288773 .042564
tau1981					
D1.	-.0229064	.0265084	-0.86	0.388	-.0749678 .029155
tau1982					
D1.	-.0495964	.0317921	-1.56	0.119	-.1120348 .012842
tau1983					
D1.	-.0591959	.0373131	-1.59	0.113	-.1324773 .0140856
tau1984					
D1.	-.0529525	.0419136	-1.26	0.207	-.1352691 .0293641

. abar, lags(2)

Arellano-Bond test for AR(1): z = -1.56 Pr > z = 0.1176

Arellano-Bond test for AR(2): z = -0.16 Pr > z = 0.8767

. ivreg2 d.n dl.n dl2.n d.w dl.w d.k dl.k dl2.k d.y d.y dl.y d.(tau1979 tau1980 tau1981
tau1982 tau1983 tau1984), noconst cluster(id)

OLS regression with robust standard errors

Number of clusters (id)	=	140	Number of obs	=	611
			F(16, 139)	=	42.36
			Prob > F	=	0.0000
Total (centered) SS	=	12.59997848	Centered R2	=	0.4363
Total (uncentered) SS	=	15.45414725	Uncentered R2	=	0.5404
Residual SS	=	7.102963775	Root MSE	=	.1078

D.n	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
n					
LD.	.1218396	.0470986	2.59	0.010	.0295282 .2141511
L2D.	-.0556518	.0393147	-1.42	0.157	-.1327073 .0214036
w					
D1.	-.5551606	.160396	-3.46	0.001	-.869531 -.2407901
LD.	.1238423	.0690606	1.79	0.073	-.0115139 .2591985
k					
D1.	.3938555	.0577114	6.82	0.000	.2807432 .5069678
LD.	.1252152	.0400739	3.12	0.002	.0466719 .2037585

L2D.	.040182	.0294136	1.37	0.172	-.0174675	.0978315
ys						
D1.	.5692903	.1675732	3.40	0.001	.2408529	.8977277
LD.	-.3754761	.1494496	-2.51	0.012	-.6683919	-.0825604
L2D.	.0272238	.1210112	0.22	0.822	-.2099538	.2644014
tau1979						
D1.	-.0000954	.0089401	-0.01	0.991	-.0176177	.0174268
tau1980						
D1.	.0068434	.0167252	0.41	0.682	-.0259374	.0396241
tau1981						
D1.	-.0229064	.0299926	-0.76	0.445	-.0816907	.0358779
tau1982						
D1.	-.0495964	.0360871	-1.37	0.169	-.1203258	.021133
tau1983						
D1.	-.0591959	.0418965	-1.41	0.158	-.1413115	.0229198
tau1984						
D1.	-.0529525	.0489099	-1.08	0.279	-.1488142	.0429092

. abar, lags(2)

Arellano-Bond test for AR(1): z = -1.51 Pr > z = 0.1305

Arellano-Bond test for AR(2): z = -0.14 Pr > z = 0.8879

.
. xtabond2 n l(1/2).n l(0/1).w l(0/2).(k ys) tau1979-tau1984, iv(l(1/2).n l(0/1).w l(0/2).(k ys) tau1979-tau1984, eq(diff)) noleveleq robust
Favoring speed over space. To switch, type or click on mata: mata set matafavor space.

Arellano-Bond dynamic panel-data estimation, one-step difference GMM results

Group variable: id	Number of obs	=	611
Time variable : year	Number of groups	=	140
Number of instruments = 16	Obs per group: min	=	4
Wald chi2(16) = 699.92	avg	=	4.36
Prob > chi2 = 0.000	max	=	6

	Robust				
	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
n					
L1.	.1218396	.0470986	2.59	0.010	.0295282 .2141511
L2.	-.0556518	.0393147	-1.42	0.157	-.1327073 .0214036
w					
--.	-.5551606	.160396	-3.46	0.001	-.869531 -.2407901
L1.	.1238423	.0690606	1.79	0.073	-.0115139 .2591985
k					
--.	.3938555	.0577114	6.82	0.000	.2807432 .5069678
L1.	.1252152	.0400739	3.12	0.002	.0466719 .2037585
L2.	.040182	.0294136	1.37	0.172	-.0174675 .0978315
ys					
--.	.5692903	.1675732	3.40	0.001	.2408529 .8977277
L1.	-.3754761	.1494496	-2.51	0.012	-.6683919 -.0825604
L2.	.0272238	.1210112	0.22	0.822	-.2099538 .2644014
tau1979					
D1.	-.0000954	.0089401	-0.01	0.991	-.0176177 .0174268
tau1980					
D1.	.0068434	.0167252	0.41	0.682	-.0259374 .0396241
tau1981					
D1.	-.0229064	.0299926	-0.76	0.445	-.0816907 .0358779
tau1982					
D1.	-.0495964	.0360871	-1.37	0.169	-.1203258 .021133
tau1983					
D1.	-.0591959	.0418965	-1.41	0.158	-.1413115 .0229198
tau1984					
D1.	-.0529525	.0489099	-1.08	0.279	-.1488142 .0429092

Hansen test of overid. restrictions: chi2(0) = 0.00 Prob > chi2 = .

Arellano-Bond test for AR(1) in first differences: z = -1.51 Pr > z = 0.131
Arellano-Bond test for AR(2) in first differences: z = -0.14 Pr > z = 0.888

. predict yhat
(280 missing values generated)

. qui corr yhat d.n if e(sample)

```

. scalar XTAB_r2_D=r(rho)^2
. qui corr yhat n if e(sample)
. scalar XTAB_r2_L=r(rho)^2
. drop yhat
. di XTAB_r2_D
.01403893
. di XTAB_r2_L
.86391605

```

Considerations

- Estimating in first-differences implies the loss of another year (1978). Temporal dummy 1979 is included because, by definition, the constant is differenced out. In fact, the inclusion of a constant would be equivalent to including a trend as a variable in the levels model:

$$y_{it} = a + b y_{it-1} + c x_{it} + \varepsilon_{it} \Rightarrow \Delta y_{it} = b \Delta y_{it-1} + c \Delta x_{it} + \Delta \varepsilon_{it}$$

$$y_{it} = \alpha t + \beta y_{it-1} + \gamma x_{it} + \eta_{it} \Rightarrow \Delta y_{it} = \alpha + \beta \Delta y_{it-1} + \gamma \Delta x_{it} + \Delta \eta_{it}.$$

In `xtabond2` with the suboption `eq(diff)` we use equations in first-differences instrumented by first-differences only. Note that it is better to add the option `nolevels eq`, otherwise equation in levels are implicitly added (GMM-sys) and the number of observations is 751 rather than 611. With these option (and unlike the old `xtabond`, or DPD, or `xtabond2`, `eq(level)`, or `xtivreg`, or `ivreg2`, in which the option `noconstant` is to be used) the constant is automatically differenced out.

- Despite it is appropriate to run the autocorrelation tests, `abar.ado` does not work after `xtivreg`, `fd`. This command understands time-series operators, but does not allow for robust/cluster standard errors even in Stata 9 (but bootstrapping is allowed). It is to be noted that it transforms in first-differences all the variables, exogenous, endogenous, dummies, and the instruments.
- The `ivreg2`, `small` command perfectly reproduces `xtivreg`, `fd`. Thus, it can be used to obtain first-differenced estimates with robust and asymptotic standard errors. The same is true for `xtabond2`. Note that the first-differences transformation has to be explicitly written in the `ivreg2` command, while it is automatically produced by `xtabond2`, `eq(diff)` that also keeps the original names of the variables. The command `xtabond2` could also work as the command `ivreg2`: by using the options `eq(lev)` `ar1e` and by writing all the variables in first-differences.

General consideration about OLS, FE and FD estimation of a dynamic panel model.

- OLS pooled estimate of lagged dependent variable is upwards biased because the same constant is imposed to all the individuals.
- The F test in the fixed effects estimate without `cluster` option shows that significant firm-specific effects exist.
- However, also fixed effects estimate is inconsistent as $N \rightarrow \infty$ if T is kept fixed.¹⁵ This bias is due to the correlation between lagged dependent variable and error term.
- The bias of estimates is even more evident when first difference transformation are used.

3.4. Anderson-Hsiao (1981)¹⁶ estimates with IV in differences: Arellano-Bond (1991), Table 5, col. (e), p. 292

¹⁵ Nickell S. (1981) Biases in dynamic models with fixed effects, *Econometrica*, 49, 6, 1417-1426.

¹⁶ Anderson T.W., Hsiao C. (1981), Estimation of dynamic models with error components, *Journal of the American Statistical Association*, 76, 598-606.

. xtivreg n (l.n l2.n l(0/1).w l(0/2).(k ys) tau1980-tau1984 = 13.n l2.n l(0/1).w l(0/2).(k ys) tau1980-tau1984), fd noconst

First-differenced IV regression
Group variable: id

R-sq: within = 0.0121
between = 0.9178
overall = 0.9895

corr(u_i, Xb) = 0.9270

Number of obs = 471
Number of groups = 140

Obs per group: min = 3
avg = 3.4
max = 5

chi2(15) = 223.84
Prob > chi2 = 0.0000

d.n	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
<hr/>					
n					
LD. 1.422765	1.583053	0.90	0.369	-1.679962	4.525493
L2D. -.1645517	.1647179	-1.00	0.318	-.4873928	.1582894
<hr/>					
w					
D1. -.7524675	.1765733	-4.26	0.000	-1.098545	-.4063902
LD. .9627611	1.086506	0.89	0.376	-1.166752	3.092275
k					
D1. .3221686	.1466086	2.20	0.028	.0348211	.6095161
LD. -.3248778	.5800599	-0.56	0.575	-1.461774	.8120187
L2D. -.0953947	.1960883	-0.49	0.627	-.4797207	.2889314
ys					
D1. .7660906	.369694	2.07	0.038	.0415037	1.490678
LD. -1.361881	1.156835	-1.18	0.239	-3.629237	.9054744
L2D. .3212993	.5440403	0.59	0.555	-.745	1.387599
tau1980					
D1. .0161204	.0336264	0.48	0.632	-.0497861	.082027
tau1981					
D1. -.0251788	.0480382	-0.52	0.600	-.1193319	.0689742
tau1982					
D1. -.0399339	.0614387	-0.65	0.516	-.1603515	.0804838
tau1983					
D1. -.0418336	.0730074	-0.57	0.567	-.1849255	.1012584
tau1984					
D1. -.0366085	.0774638	-0.47	0.637	-.1884349	.1152178
<hr/>					
sigma_u .29195156					
sigma_e .18855982					
rho .70564904	(fraction of variance due to u_i)				
<hr/>					

Instrumented: L.n L2.n w L.w k L.k L2.k ys L.ys L2.ys tau1980 tau1981 tau1982 tau1983
tau1984

Instruments: L3.n L2.n w L.w k L.k L2.k ys L.ys L2.ys tau1980 tau1981 tau1982 tau1983
tau1984

. xtabond2 n l(1/2).n l(0/1).w l(0/2).(k ys) tau1980-tau1984, iv(l(2/3).n l(0/1).w l(0/2).(k ys) tau1980-tau1984, eq(diff)) robust noleveleq
Favoring speed over space. To switch, type or click on mata: mata set matafavor space.

Arellano-Bond dynamic panel-data estimation, one-step difference GMM results

Group variable: id	Number of obs = 471
Time variable : year	Number of groups = 140
Number of instruments = 15	Obs per group: min = 3
Wald chi2(15) = 836.36	avg = 3.36
Prob > chi2 = 0.000	max = 5
<hr/>	

	Robust				
	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
<hr/>					
n					
L1. 1.422765	1.001091	1.42	0.155	-.5393375	3.384868
L2. -.1645517	.1276498	-1.29	0.197	-.4147407	.0856372
w					
--. -.7524675	.229792	-3.27	0.001	-1.202852	-.3020834

```

L1. | .9627611 .7683295 1.25 0.210 -.5431371 2.468659
k |
--. | .3221686 .104688 3.08 0.002 .1169839 .5273533
L1. | -.3248779 .3860559 -0.84 0.400 -1.081534 .4317779
L2. | -.0953947 .1234367 -0.77 0.440 -.3373261 .1465368
ys |
--. | .7660906 .3113873 2.46 0.014 .1557827 1.376399
L1. | -1.361881 .8814085 -1.55 0.122 -3.08941 .3656478
L2. | .3212993 .4156362 0.77 0.440 -.4933326 1.135931
tau1980 | .0161204 .0249057 0.65 0.517 -.0326939 .0649348
tau1981 | -.0251788 .037007 -0.68 0.496 -.0977112 .0473535
tau1982 | -.0399339 .0396221 -1.01 0.314 -.1175918 .037724
tau1983 | -.0418336 .0434498 -0.96 0.336 -.1269936 .0433265
tau1984 | -.0366085 .0475629 -0.77 0.441 -.1298301 .056613
-----
Hansen test of overid. restrictions: chi2(0) = 0.00 Prob > chi2 =
Arellano-Bond test for AR(1) in first differences: z = -1.24 Pr > z = 0.216
Arellano-Bond test for AR(2) in first differences: z = -0.78 Pr > z = 0.435
-----
```

Considerations.

- Instrumenting first-differences of y_{it-1} with Δy_{it-3} ¹⁷ implies the lost of another year (1979). The correct specification is to drop the constant and to include the 1980 temporal dummy. Again, the only estimation command in which we do not need to use the option `noconstant` is `xtabond2 eq(diff)`.
- From now on it is important to test H_0 : errors not correlated at the second order i.e. dynamics correctly specified. Of course, H_0 : errors not correlated at the first order is always rejected because in the first difference equations errors $\sim MA(1)$.

3.5. Anderson-Hsiao (1982)¹⁸ estimates with IV in levels: Arellano-Bond (1991), Table 5, col. (f), p. 292

```

. facum id n
(140 missing values generated)
(140 real changes made)

. xtivreg n (l.n l2.n l(0/1).w l(0/2).(k ys) tau1979-tau1984 = 12.cumnl l2.n l(0/1).w
l(0/2).(k ys) tau1979-tau1984), noconst fd

First-differenced IV regression
Group variable: id
Number of obs = 611
Number of groups = 140

R-sq: within = 0.0561
      between = 0.8740
      overall = 0.9615
Obs per group: min = 4
                  avg = 4.4
                  max = 6

corr(u_i, Xb) = -0.7735
chi2(16) = 133.13
Prob > chi2 = 0.0000
-----

d.n | Coef. Std. Err. z P>|z| [95% Conf. Interval]
-----+
n |
LD. | 2.307626 1.999548 1.15 0.248 -1.611416 6.226668
L2D. | -.2240271 .1814343 -1.23 0.217 -.5796318 .1315777
w |
D1. | -.8103626 .2653018 -3.05 0.002 -1.330345 -.2903807
-----
```

¹⁷ The IV-set is composed by all the exogenous variables of the system (both included and excluded from the equation of interest): hence, Δy_{it-2} , Δy_{it-3} , plus contemporaneous and lagged first-differences of w, k, ys. Given that Δy_{it-2} is an exogenous included in the equation, the actual IV for y_{it-1} is Δy_{it-3} .

¹⁸ Anderson T.W., Hsiao C. (1982), Formulation and Estimation of dynamic models using panel data, *Journal of Econometrics*, 18, 47-82.

LD.	1.422246	1.195245	1.19	0.234	-.9203922	3.764884
k						
D1.	.2530975	.1466736	1.73	0.084	-.0343774	.5405725
LD.	-.5524614	.6237136	-0.89	0.376	-1.774918	.6699949
L2D.	-.2126364	.2429936	-0.88	0.382	-.6888952	.2636224
ys						
D1.	.9905803	.4691946	2.11	0.035	.0709758	1.910185
LD.	-1.937912	1.457434	-1.33	0.184	-4.794431	.9186063
L2D.	.4870838	.5167524	0.94	0.346	-.5257324	1.4999
tau1979						
D1.	.0626485	.0640965	0.98	0.328	-.0629784	.1882754
tau1980						
D1.	.1080019	.1013824	1.07	0.287	-.090704	.3067078
tau1981						
D1.	.0704241	.1046861	0.67	0.501	-.1347568	.2756051
tau1982						
D1.	.076515	.1363112	0.56	0.575	-.1906502	.3436801
tau1983						
D1.	.0895758	.1605703	0.56	0.577	-.2251361	.4042878
tau1984						
D1.	.0956024	.1662583	0.58	0.565	-.2302579	.4214626
-----	-----	-----	-----	-----	-----	-----
sigma_u	.33809698					
sigma_e	.25030024					
rho	.64596356	(fraction of variance due to u_i)				
-----	-----	-----	-----	-----	-----	-----
Instrumented:	L.n L2.n w L.w k L.k L2.k ys L.y s L2.y s tau1979 tau1980 tau1981 tau1982					
tau1983 tau1984						
Instruments:	L2.cumnl L2.n w L.w k L.k L2.k ys L.y s L2.y s tau1979 tau1980 tau1981 tau1982					
tau1983						
tau1984						
-----	-----	-----	-----	-----	-----	-----
. xtabond2 n l(1/2).n l(0/1).w l(0/2).(k ys) tau1979-tau1984, iv(l2.n l(0/1).w l(0/2).(k ys)						
tau1979-tau1984, eq(diff)) iv(l3.n , eq(diff) passthru) robust noleveleq						
Favoring speed over space. To switch, type or click on mata: mata set matafavor space.						
Arellano-Bond dynamic panel-data estimation, one-step difference GMM results						
-----	-----	-----	-----	-----	-----	-----
Group variable: id					Number of obs =	611
Time variable : year					Number of groups =	140
Number of instruments = 16					Obs per group: min =	4
Wald chi2(16) = 302.91					avg =	4.36
Prob > chi2 = 0.000					max =	6
-----	-----	-----	-----	-----	-----	-----
	Robust					
	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
-----	-----	-----	-----	-----	-----	-----
n						
L1.	2.307626	1.054549	2.19	0.029	.2407485	4.374503
L2.	-.2240271	.1172406	-1.91	0.056	-.4538144	.0057602
w						
--.	-.8103626	.283096	-2.86	0.004	-1.365221	-.2555047
L1.	1.422246	.8507098	1.67	0.095	-.2451148	3.089606
k						
--.	.2530975	.1103882	2.29	0.022	.0367407	.4694543
L1.	-.5524613	.3572037	-1.55	0.122	-1.252568	.147645
L2.	-.2126364	.1453826	-1.46	0.144	-.4975811	.0723083
ys						
--.	.9905803	.3376145	2.93	0.003	.3288681	1.652293
L1.	-1.937912	.9923162	-1.95	0.051	-3.882816	.006992
L2.	.4870838	.4247408	1.15	0.251	-.3453929	1.31956
tau1979						
D1.	.0626485	.0333423	1.88	0.060	-.0027012	.1279982
tau1980						
D1.	.1080019	.0536941	2.01	0.044	.0027633	.2132405
tau1981						
D1.	.0704241	.0632658	1.11	0.266	-.0535746	.1944228
tau1982						
D1.	.076515	.0697354	1.10	0.273	-.0601639	.2131938
tau1983						
D1.	.0895758	.0821999	1.09	0.276	-.071533	.2506847
tau1984						
D1.	.0956023	.0780306	1.23	0.221	-.0573348	.2485395
-----	-----	-----	-----	-----	-----	-----
Hansen test of overid. restrictions: chi2(0) = 0.00					Prob > chi2 = .	

```

Arellano-Bond test for AR(1) in first differences: z = -1.98 Pr > z = 0.048
Arellano-Bond test for AR(2) in first differences: z = -0.92 Pr > z = 0.358
-----
```

Considerations:

- Estimating in first-differences and instrumentint Δy_{it-1} with y_{it-3} ¹⁹ implies that the year 1979 is saved. Thus, the correct specification is to drop the constant (the model is in first differences so that the constant is differenced out) and to include 1979 temporal dummy.
- The command `xtivreg, fd` needs the `facum.ado`, a procedure that we wrote in order to preserve the levels of the instruments despite the first-differencing: the trick is to create the cumulated of the involved variable; it is lagged, explaining why, in the estimation command, we need to put lag t-2, instead of lag t-3. This procedure also replace initial missing with zero.
- The option `passthru` with `eq(diff)` in `xtabond2` leaves IVs in levels for the first-differences equations. There is also an option `mz` that, if added to the `ivstyle`, convertes missing values in the IVs into zeroes. In this context, however, this option is irrelevant, because results do not change.
- Until now we are estimating a model exactly identified, with IV (2SLS). Hence, the overidentification tests do not work; onestep and twostep coincide; no estimation of the variance-covariance matrix of the error is done, i.e. the MA(1) structure in the error term induced by the first-differencing is not taken into account; the choice of `h(.)`, an option in `xtabond2` that selects the a priori estimate of the variance-covariance matrix of the error terms, is irrelevant.
- The AndersonHsiao estimates are less precise if compared to the GMM ones because they do not use all the relevant information about the structure of the error term, nor all the available moment conditions.

3.6. GMM-dif estimates: Arellano-Bond (1991), Table 4, col. (a1) (a2), p. 290

```

. xtabond2 n l(1/2).n l(0/1).w l(0/2).(k ys) tau1979-tau1984, iv(l(0/1).w l(0/2).(k ys)
tau1979-tau1984, eq(diff)) gmm(n, laglimits(2 .) eq(diff)) noleveleq h(2)
Favoring speed over space. To switch, type or click on mata: mata set matafavor space.

```

Arellano-Bond dynamic panel-data estimation, one-step difference GMM results

Group variable: id				Number of obs	=	611
Time variable : year				Number of groups	=	140
Number of instruments = 41				Obs per group: min	=	4
Wald chi2(16) = 1804.32				avg	=	4.36
Prob > chi2 = 0.000				max	=	6
			Coef.	Std. Err.	z	P> z
		+				[95% Conf. Interval]
		n				
L1.	.6862261	.1466575	4.68	0.000	.3987827	.9736696
L2.	-.0853582	.0438509	-1.95	0.052	-.1713043	.0005879
w						
--.	-.6078208	.0649026	-9.37	0.000	-.7350275	-.4806141
L1.	.3926237	.1077977	3.64	0.000	.1813441	.6039032
k						
--.	.3568456	.0365434	9.76	0.000	.2852219	.4284693
L1.	-.0580012	.0575366	-1.01	0.313	-.1707708	.0547685
L2.	-.0199475	.0410788	-0.49	0.627	-.1004604	.0605654
ys						
--.	.6085073	.1327679	4.58	0.000	.348287	.8687276
L1.	-.7111651	.1820286	-3.91	0.000	-1.067935	-.3543955
L2.	.1057969	.140974	0.75	0.453	-.170507	.3821008
tau1979	.0095545	.01402	0.68	0.496	-.0179242	.0370332
tau1980	.0220152	.0183075	1.20	0.229	-.0138669	.0578973
tau1981	-.0117743	.0242915	-0.48	0.628	-.0593848	.0358362

¹⁹ The IV-set is composed by all the exogenous variables of the system (both included and excluded from the equation of interest): hence, y_{it-2} , y_{it-3} , plus contemporaneous and lagged levels of w, k, ys. Given that y_{it-2} is an exogenous included in the equation, the actual IV for y_{it-1} is y_{it-3} .

tau1982 | -.0270588 .0256089 -1.06 0.291 -.0772514 .0231338
 tau1983 | -.0213204 .0279544 -0.76 0.446 -.07611 .0334691
 tau1984 | -.0077033 .0304006 -0.25 0.800 -.0672875 .0518808

 Sargan test of overid. restrictions: chi2(26) = 67.59 Prob > chi2 = 0.000
 Arellano-Bond test for AR(1) in first differences: z = -3.99 Pr > z = 0.000
 Arellano-Bond test for AR(2) in first differences: z = -0.55 Pr > z = 0.583

 . xtabond2 n l(1/2).n l(0/1).w l(0/2).(k ys) tau1979-tau1984, iv(l(0/1).w l(0/2).(k ys) tau1979-tau1984, eq(diff)) gmm(n, laglimits(2 .) eq(diff)) robust noleveleq h(2)
 Favoring speed over space. To switch, type or click on mata: mata set matafavor space.
 Arellano-Bond dynamic panel-data estimation, one-step difference GMM results

 Group variable: id Number of obs = 611
 Time variable : year Number of groups = 140
 Number of instruments = 41 Obs per group: min = 4
 Wald chi2(16) = 1727.45 avg = 4.36
 Prob > chi2 = 0.000 max = 6

 | Robust
 | Coef. Std. Err. z P>|z| [95% Conf. Interval]
 +-----
 n |
 L1. | .6862261 .1445943 4.75 0.000 .4028266 .9696257
 L2. | -.0853582 .0560155 -1.52 0.128 -.1951467 .0244302
 w |
 --. | -.6078208 .1782055 -3.41 0.001 -.9570972 -.2585445
 L1. | .3926237 .1679931 2.34 0.019 .0633632 .7218842
 k |
 --. | .3568456 .0590203 6.05 0.000 .241168 .4725233
 L1. | -.0580012 .0731797 -0.79 0.428 -.2014308 .0854284
 L2. | -.0199475 .0327126 -0.61 0.542 -.0840631 .0441681
 ys |
 --. | .6085073 .1725313 3.53 0.000 .2703522 .9466624
 L1. | -.7111651 .2317163 -3.07 0.002 -1.165321 -.2570095
 L2. | .1057969 .1412021 0.75 0.454 -.1709542 .382548
 tau1979 | .0095545 .0102896 0.93 0.353 -.0106127 .0297217
 tau1980 | .0220152 .0177104 1.24 0.214 -.0126966 .056727
 tau1981 | -.0117743 .0295079 -0.40 0.690 -.0696086 .04606
 tau1982 | -.0270588 .0292751 -0.92 0.355 -.0844369 .0303193
 tau1983 | -.0213204 .0304599 -0.70 0.484 -.0810207 .0383798
 tau1984 | -.0077033 .0314106 -0.25 0.806 -.069267 .0538604

 Hansen test of overid. restrictions: chi2(26) = 31.38 Prob > chi2 = 0.214
 Arellano-Bond test for AR(1) in first differences: z = -3.60 Pr > z = 0.000
 Arellano-Bond test for AR(2) in first differences: z = -0.52 Pr > z = 0.606

 . xtabond2 n l(1/2).n l(0/1).w l(0/2).(k ys) tau1979-tau1984, iv(l(0/1).w l(0/2).(k ys) tau1979-tau1984, eq(diff)) gmm(n, laglimits(2 .) eq(diff)) robust noleveleq h(2) twostep
 Favoring speed over space. To switch, type or click on mata: mata set matafavor space.
 Arellano-Bond dynamic panel-data estimation, two-step difference GMM results

 Group variable: id Number of obs = 611
 Time variable : year Number of groups = 140
 Number of instruments = 41 Obs per group: min = 4
 Wald chi2(16) = 1104.72 avg = 4.36
 Prob > chi2 = 0.000 max = 6

 | Corrected
 | Coef. Std. Err. z P>|z| [95% Conf. Interval]
 +-----
 n |
 L1. | .6287089 .1934138 3.25 0.001 .2496248 1.007793
 L2. | -.0651882 .0450501 -1.45 0.148 -.1534847 .0231084

w						
--.	-.5257597	.1546107	-3.40	0.001	-.828791	-.2227284
L1.	.3112899	.2030006	1.53	0.125	-.086584	.7091638
k						
--.	.2783619	.0728019	3.82	0.000	.1356728	.4210511
L1.	.0140994	.0924575	0.15	0.879	-.167114	.1953129
L2.	-.0402484	.0432745	-0.93	0.352	-.1250649	.0445681
ys						
--.	.5919243	.1730916	3.42	0.001	.252671	.9311776
L1.	-.5659863	.2611008	-2.17	0.030	-1.077734	-.0542381
L2.	.1005433	.1610987	0.62	0.533	-.2152043	.4162908
tau1979	.0112155	.0116783	0.96	0.337	-.0116735	.0341045
tau1980	.0230688	.020056	1.15	0.250	-.0162402	.0623778
tau1981	-.0213579	.0332439	-0.64	0.521	-.0865147	.0437989
tau1982	-.0311159	.0339723	-0.92	0.360	-.0977005	.0354686
tau1983	-.0179932	.0369328	-0.49	0.626	-.0903802	.0543938
tau1984	-.0233675	.0366145	-0.64	0.523	-.0951306	.0483956

Hansen test of overid. restrictions: chi2(26) = 31.38 Prob > chi2 = 0.214

Arellano-Bond test for AR(1) in first differences: z = -2.13 Pr > z = 0.034
 Arellano-Bond test for AR(2) in first differences: z = -0.35 Pr > z = 0.725

. xtabond2 n l(1/2).n l(0/1).w l(0/2).(k ys) tau1979-tau1984, iv(l(0/1).w l(0/2).(k ys) tau1979-tau1984, eq(diff)) gmm(n, laglimits(2 .) eq(diff)) noleveleq h(2) twostep
 Favoring speed over space. To switch, type or click on mata: mata set matafavor space.

Arellano-Bond dynamic panel-data estimation, two-step difference GMM results

Group variable: id					Number of obs	=	611
Time variable : year					Number of groups	=	140
Number of instruments = 41					Obs per group: min	=	4
Wald chi2(16) = 2216.93					avg	=	4.36
Prob > chi2 = 0.000					max	=	6

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
n					
L1.	.6287089	.0904543	6.95	0.000	.4514216 .8059961
L2.	-.0651882	.0265009	-2.46	0.014	-.117129 -.0132474
w					
--.	-.5257597	.0537692	-9.78	0.000	-.6311453 -.420374
L1.	.3112899	.0940116	3.31	0.001	.1270305 .4955492
k					
--.	.2783619	.0449083	6.20	0.000	.1903432 .3663807
L1.	.0140994	.0528046	0.27	0.789	-.0893957 .1175946
L2.	-.0402484	.0258038	-1.56	0.119	-.0908229 .010326
ys					
--.	.5919243	.1162114	5.09	0.000	.3641542 .8196943
L1.	-.5659863	.1396738	-4.05	0.000	-.8397419 -.2922306
L2.	.1005433	.1126749	0.89	0.372	-.1202955 .321382
tau1979	.0112155	.0077507	1.45	0.148	-.0039756 .0264066
tau1980	.0230688	.0136626	1.69	0.091	-.0037094 .0498471
tau1981	-.0213579	.0224104	-0.95	0.341	-.0652815 .0225657
tau1982	-.0311159	.0231606	-1.34	0.179	-.0765099 .014278
tau1983	-.0179932	.0232122	-0.78	0.438	-.0634883 .027502
tau1984	-.0233675	.0235452	-0.99	0.321	-.0695153 .0227803

Warning: Uncorrected two-step standard errors are unreliable.

Hansen test of overid. restrictions: chi2(26) = 31.38 Prob > chi2 = 0.214

Arellano-Bond test for AR(1) in first differences: z = -3.00 Pr > z = 0.003
 Arellano-Bond test for AR(2) in first differences: z = -0.42 Pr > z = 0.678

Considerations.

- All the explanatory variables different from the lagged dependent variable are assumed to be strictly exogenous i.e. $E(\varepsilon_{it} | x_{is})=0 \forall t,s = 1, \dots, T, \forall i=1, \dots, N$.
- Column (a) reports robust standard error and homoskedastic Sargan test; hence, it is a mixture of the two first GMM-dif one-step estimates previously reported. In the one-step, the heteroskedastic Hansen test is reported when the option `robust` is specified.
- Column (b) is the fourth GMM-dif estimate, without `robust` option. Note that, in `twostep robust` (third GMM-dif estimate), the Windmeijer (2005) small sample correction for the variance of linear two-step GMM estimators is computed. Warning: do not use it in panel of not negligible size! Note also that, in the two-step, the heteroskedastic Hansen test is always reported, independently from the use or not of the option `robust`.
- The `gmm` option specifies a set of variables to be used as bases for “GMM-style” instrument sets described in Holtz-Eakin, Newey, and Rosen (1988)²⁰ and Arellano-Bond (1991). By default `xtabond2` uses, for each time period, all available lags of the specified variables in levels dated t-1 or earlier as instruments for the first-difference equations, and the contemporaneous first differences as instruments in the levels equations.²¹ The suboption `laglimits(a b)` can override these defaults: for the first-difference equations, lagged levels dated $t-a$ to $t-b$ are used as instruments, while for the levels equations, the first-difference dated $t-a+1$ is normally used. Note that a and b can each be missing (“.”) intending infinity; they can even be negative, implying “forward” lags. Hence, there are different ways of writing `gmm` for `eq(diff)`. For example, the use of y_{it-2} and as an IV for y_{it-1} in equation in first-differences can be written as: `gmm(y, laglimits(2 .))`, or `gmm(12.y, laglimits(0 .))`, or `gmm(1.y, laglimits(1 .))` or, the default, `gmm(1.y)`.
- In `gmm` the moment conditions are imposed for each time period, variable and lag distance. For example, when we use `lag(2 2)`, we have 5 over-identifying restrictions:

Equations *IVs*

$$\begin{aligned} y_{i1979} &= \beta_1 y_{i1978} + \beta_2 y_{i1977} & y_{i1977} \\ y_{i1980} &= \beta_1 y_{i1979} + \beta_2 y_{i1978} & y_{i1978} \\ y_{i1981} &= \beta_1 y_{i1980} + \beta_2 y_{i1979} & y_{i1979} \\ y_{i1982} &= \beta_1 y_{i1981} + \beta_2 y_{i1980} & y_{i1980} \\ y_{i1983} &= \beta_1 y_{i1982} + \beta_2 y_{i1981} & y_{i1981} \\ y_{i1984} &= \beta_1 y_{i1983} + \beta_2 y_{i1982} & y_{i1982} \end{aligned}$$

For 1 endogenous variable, y_{it-1} , 6 instruments, y_{it-2} , are available (one in each equation, from 1979 to 1984).²²

If we use `lag(2 3)`, we have 11 over-identifying restrictions:

Equations *IVs*

$$\begin{aligned} y_{i1979} &= \beta_1 y_{i1978} + \beta_2 y_{i1977} & y_{i1976} y_{i1977} \\ y_{i1980} &= \beta_1 y_{i1979} + \beta_2 y_{i1978} & y_{i1977} y_{i1978} \\ y_{i1981} &= \beta_1 y_{i1980} + \beta_2 y_{i1979} & y_{i1978} y_{i1979} \\ y_{i1982} &= \beta_1 y_{i1981} + \beta_2 y_{i1980} & y_{i1979} y_{i1980} \\ y_{i1983} &= \beta_1 y_{i1982} + \beta_2 y_{i1981} & y_{i1980} y_{i1981} \\ y_{i1984} &= \beta_1 y_{i1983} + \beta_2 y_{i1982} & y_{i1981} y_{i1982} \end{aligned}$$

For 1 endogenous variable, y_{it-1} , 12 instruments, y_{it-2} and y_{it-3} , are available (two in each equation from 1979 to 1984).²³

²⁰ Holtz-Eakin, D., W. Newey, and H.S. Rosen (1988) Estimating vector autoregressions with panel data, *Econometrica*, 56, 1371-95.

²¹ These defaults are appropriate for predetermined variables (Bond 2000).

²² Note that, in the Anderson-Hsiao IV estimation method, where one instrument is imposed to each variable and lag distance, y_{it-2} alone is not able to identify the equation of interest because it is also the included exogenous.

Finally, if we use lag(2 .), we have 26 over-identifying restrictions:

Equations

IVs

$$\begin{array}{ll}
 y_{i1979} = \beta_1 y_{i1978} + \beta_2 y_{i1977} & y_{i1976} y_{1977} \\
 y_{i1980} = \beta_1 y_{i1979} + \beta_2 y_{i1978} & y_{i1976} y_{i1977} y_{1978} \\
 y_{i1981} = \beta_1 y_{i1980} + \beta_2 y_{i1979} & y_{i1976} y_{i1977} y_{i1978} y_{1979} \\
 y_{i1982} = \beta_1 y_{i1981} + \beta_2 y_{i1980} & y_{i1976} y_{i1977} y_{i1978} y_{i1979} y_{1980} \\
 y_{i1983} = \beta_1 y_{i1982} + \beta_2 y_{i1981} & y_{i1976} y_{i1977} y_{i1978} y_{i1979} y_{i1980} y_{1981} \\
 y_{i1984} = \beta_1 y_{i1983} + \beta_2 y_{i1982} & y_{i1976} y_{i1977} y_{i1978} y_{i1979} y_{i1980} y_{i1981} y_{1982}
 \end{array}$$

For 1 endogenous variable, y_{it-1} , we have 27 moment conditions available, y_{it-2} , y_{it-3} , y_{it-4} , ..., y_{i0} (from two in the first equation of 1979, to three in the second equation of 1980, to all the seven available lags in the last equation of 1984).²⁴

- In GMM, by default, missing values are always replaced by zeros.
- When we use `xtabond2` with `ivstyle` only (or with `gmm collapse`) we need only onestep: twostep is irrelevant because no estimation of the covariance matrix of the error is done; hence, also the choice of $h(.)$, the a priori estimate of the covariance matrix of the error terms that serves as the first-step inner product matrix to weight the sample moments whose magnitudes are minimised, is irrelevant. Matrix $h(.)$ always has a block diagonal form, with all the $T \times T$ blocks the same. When we use `xtabond2 gmm` the choice of $h(.)$ is relevant: the default, $h(3)$, will be discussed in section 3.8 on GMM-sys. Option $h(1)$ selects the identity matrix used to produce IV/2SLS. Option $h(2)$ accounts for the MA(1) structure of the error term in the first-differenced model: the blocks of H are like

$$\begin{matrix}
 2 & -1 & 0 & \dots \\
 -1 & 2 & -1 & \dots \\
 0 & -1 & 2 & \dots
 \end{matrix}$$

Consider the error term $v_{it} = \mu_i + \varepsilon_{it}$. If the VCOV matrix of the error term, $E(\varepsilon\varepsilon')$, is the identity matrix, i.e. ε is homoskedastic, in the first-differenced equations we have the following VCOV matrix: $E(\Delta v \Delta v') = E(\Delta \varepsilon \Delta \varepsilon') = \text{Var}(\varepsilon_{it} - \varepsilon_{it-1}) = \text{Var}(\varepsilon_{it}) - 2\text{Cov}(\varepsilon_{it}, \varepsilon_{it-1}) + \text{Var}(\varepsilon_{it-1}) = 1 - 2*0 + 1$. Hence, $\text{Var}(\Delta \varepsilon_{it})$, a diagonal element of the error covariance matrix, is 2; similarly, $\text{Cov}(\Delta \varepsilon_{it}, \Delta \varepsilon_{it-1})$, on the first subdiagonal of the error covariance matrix, works out to equal -1.

3.7. GMM-dif estimates: Arellano-Bond (1991), Table 4, col. (b) (c), p. 290

```
. xtabond2 n 1(1/2).n 1(0/1).w k 1(0/2).ys tau1979-tau1984, iv(1(0/1).w k 1(0/2).ys tau1979-tau1984, eq(diff)) gmm(n, laglimits(2 .) eq(diff)) noleveleq h(2) twostep
Favoring speed over space. To switch, type or click on mata: mata set matafavor space.
```

Arellano-Bond dynamic panel-data estimation, two-step difference GMM results

Group variable: id	Number of obs	=	611
Time variable : year	Number of groups	=	140
Number of instruments = 39	Obs per group: min	=	4
Wald chi2(14) = 1582.82	avg	=	4.36
Prob > chi2 = 0.000	max	=	6

Coef. Std. Err. z P> z [95% Conf. Interval]			

²³ Note that, in the Anderson-Hsiao IV estimation method, y_{it-2} and y_{it-3} produce an exactly identified model, given that y_{it-2} is also the exogenous variable included in the equation of interest.

²⁴ Note that, in the Anderson-Hsiao IV estimation method, y_{it-2} , y_{it-3} , y_{it-4} , y_{it-5} , y_{it-6} , y_{it-7} and y_{it-8} produce 5 over-identifying restrictions; the moment condition linked to y_{it-2} is lost because it is also the exogenous variable included in the equation of interest. The Anderson-Hsiao IV estimation method that uses all the available lags can be easily obtained by the command `xtabond2 n 1(1/2).n 1(0/1).w 1(0/2).(k ys) tau1979-tau1984, iv(1(0/1).w 1(0/2).(k ys) tau1979-tau1984, eq(diff)) gmm(n, laglimits(2 .) eq(diff) collapse) noleveleq h(2) robust`

n						
L1. .4642432	.0862874	5.38	0.000	.2951229	.6333634	
L2. -.0566277	.0268605	-2.11	0.035	-.1092734	-.0039821	
w						
--. -.5085975	.0513303	-9.91	0.000	-.609203	-.4079919	
L1. .2254592	.0795659	2.83	0.005	.069513	.3814054	
k .2899209	.0390319	7.43	0.000	.2134198	.3664221	
ys						
--. .5889862	.1105944	5.33	0.000	.3722253	.8057472	
L1. -.445183	.1274288	-3.49	0.000	-.6949389	-.1954271	
L2. .0874253	.107306	0.81	0.415	-.1228907	.2977412	
tau1979 .0092642	.0072076	1.29	0.199	-.0048624	.0233909	
tau1980 .0214197	.0122459	1.75	0.080	-.0025817	.0454212	
tau1981 -.0217047	.0206961	-1.05	0.294	-.0622683	.018859	
tau1982 -.0402667	.0229537	-1.75	0.079	-.0852552	.0047218	
tau1983 -.0357929	.0244132	-1.47	0.143	-.083642	.0120561	
tau1984 -.0443511	.0256751	-1.73	0.084	-.0946734	.0059712	

Warning: Uncorrected two-step standard errors are unreliable.

Hansen test of overid. restrictions: chi2(26) = 29.58 Prob > chi2 = 0.285

Arellano-Bond test for AR(1) in first differences: z = -2.39 Pr > z = 0.017
Arellano-Bond test for AR(2) in first differences: z = -0.24 Pr > z = 0.808

. xtabond2 n l(1/2).n l(0/1).w k l(0/1).ys tau1979-tau1984, iv(l(0/1).ys tau1979-tau1984, eq(diff)) gmm(n, lag(2 .) eq(diff)) gmm(w k, lag(2 3) eq(diff)) noleveleq h(2) twostep
Favoring speed over space. To switch, type or click on mata: mata set matafavor space.

Arellano-Bond dynamic panel-data estimation, two-step difference GMM results

Group variable: id	Number of obs	=	611			
Time variable : year	Number of groups	=	140			
Number of instruments = 59	Obs per group: min	=	4			
Wald chi2(13) = 2668.33	avg	=	4.36			
Prob > chi2 = 0.000	max	=	6			

Coef. Std. Err. z P> z [95% Conf. Interval]						

n						
L1. .8074308	.0491071	16.44	0.000	.7111827	.9036789	
L2. -.1134945	.0228508	-4.97	0.000	-.1582813	-.0687076	
w						
--. -.5686242	.0598841	-9.50	0.000	-.6859949	-.4512536	
L1. .640707	.0672002	9.53	0.000	.508997	.772417	
k .1833987	.0567488	3.23	0.001	.0721731	.2946243	
ys						
--. .8599923	.1097787	7.83	0.000	.6448299	1.075155	
L1. -.8632569	.1101426	-7.84	0.000	-1.079132	-.6473814	
tau1979 .0162529	.006453	2.52	0.012	.0036053	.0289005	
tau1980 .0420327	.0105855	3.97	0.000	.0212854	.0627799	
tau1981 .0025669	.0162173	0.16	0.874	-.0292183	.0343522	
tau1982 -.0370388	.0186426	-1.99	0.047	-.0735776	-.0004999	
tau1983 -.0511972	.0238629	-2.15	0.032	-.0979676	-.0044267	
tau1984 -.0587939	.026186	-2.25	0.025	-.1101176	-.0074701	

Warning: Uncorrected two-step standard errors are unreliable.

Hansen test of overid. restrictions: chi2(49) = 58.71 Prob > chi2 = 0.161

Arellano-Bond test for AR(1) in first differences: z = -4.07 Pr > z = 0.000
Arellano-Bond test for AR(2) in first differences: z = -0.68 Pr > z = 0.498

Considerations.

- Column (b) just omits insignificant dynamics; only the twostep estimates are reported. Column (c) relaxes the assumption that real wage (w) and capital stock (k) are strictly exogenous and, instead,

treats them as being endogenous. $E(\varepsilon_{it}|x_{is})=0 \forall t > s$ and $E(\varepsilon_{it}|x_{is})\neq0 \forall t \leq s, \forall i=1,\dots,N, \forall t, s=1,\dots,T$. Hence, lags of w and k dated $(t-2)$ and earlier can be used as IVs for w_{it} , w_{it-1} and k_{it} . However, estimates of Column (c) can just be approximated because the available data-set lacks company's real sales and real stocks that Arellano-Bond use as additional IVs.

3.8. GMM-dif and GMM-sys estimates: Blundell-Bond (1998), Table 4, col. III and IV, p. 137

```
. xtabond2 n l.n l(0/1).(w k) tau1978-tau1984, iv(tau1978-tau1984, eq(diff)) gmm(n w k, lag(2
.) eq(diff)) noleveleq h(2) robust
Favoring speed over space. To switch, type or click on mata: mata set matafavor space.
```

Arellano-Bond dynamic panel-data estimation, one-step difference GMM results

		Robust				
	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
n						
L1.	.7074701	.0841788	8.40	0.000	.5424827	.8724576
w						
--.	-.7087965	.117102	-6.05	0.000	-.9383122	-.4792809
L1.	.5000149	.1113282	4.49	0.000	.2818157	.7182141
k						
--.	.4659776	.101044	4.61	0.000	.267935	.6640203
L1.	-.2151309	.0858525	-2.51	0.012	-.3833987	-.0468631
tau1978	.0057636	.0166077	0.35	0.729	-.0267868	.038314
tau1979	.0136366	.0193748	0.70	0.482	-.0243374	.0516106
tau1980	-.0071557	.0213479	-0.34	0.737	-.0489969	.0346855
tau1981	-.0340692	.0264327	-1.29	0.197	-.0858763	.0177379
tau1982	-.0059175	.0272325	-0.22	0.828	-.0592922	.0474573
tau1983	.0187213	.0288529	0.65	0.516	-.0378294	.075272
tau1984	.0352279	.0331578	1.06	0.288	-.0297603	.1002161

Hansen test of overid. restrictions: chi2(79) = 88.80 Prob > chi2 = 0.211

Arellano-Bond test for AR(1) in first differences: z = -5.60 Pr > z = 0.000
Arellano-Bond test for AR(2) in first differences: z = -0.14 Pr > z = 0.891

```
. xtabond2 n l.n l(0/1).(w k) tau1978-tau1984, iv(tau1978-tau1984, eq(both)) gmm(n w k, lag(2
.) eq(both)) h(1) robust
Favoring speed over space. To switch, type or click on mata: mata set matafavor space.
```

Arellano-Bond dynamic panel-data estimation, one-step system GMM results

		Robust				
	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
n						
L1.	.8108394	.0579982	13.98	0.000	.6971649	.9245138
w						
--.	-.7945394	.0971517	-8.18	0.000	-.9849532	-.6041257
L1.	.55012	.151645	3.63	0.000	.2529012	.8473388
k						
--.	.4285055	.0763361	5.61	0.000	.2788895	.5781215
L1.	-.2802184	.0776689	-3.61	0.000	-.4324466	-.1279903

tau1978		.0077488	.0200664	0.39	0.699	-.0315806	.0470781
tau1979		.020829	.0236973	0.88	0.379	-.025617	.0672749
tau1980		-.0002589	.0252166	-0.01	0.992	-.0496826	.0491648
tau1981		-.0271456	.02961	-0.92	0.359	-.0851801	.030889
tau1982		.0012306	.026954	0.05	0.964	-.0515983	.0540596
tau1983		.014436	.0254967	0.57	0.571	-.0355367	.0644087
tau1984		.0003278	.0307739	0.01	0.992	-.059988	.0606436
_cons		1.006162	.430149	2.34	0.019	.1630853	1.849238

Hansen test of overid. restrictions: chi2(106) = 115.73 Prob > chi2 = 0.244

Arellano-Bond test for AR(1) in first differences: z = -6.49 Pr > z = 0.000
 Arellano-Bond test for AR(2) in first differences: z = -0.08 Pr > z = 0.934

. xtabond2 n l.n l(0/1).(w k) tau1978-tau1984, iv(tau1978-tau1984, eq(both)) gmm(n w k, lag(2
 .) eq(both)) h(2) robust

Favoring speed over space. To switch, type or click on mata: mata set matafavor space.

Arellano-Bond dynamic panel-data estimation, one-step system GMM results

Group variable: id				Number of obs	=	891
Time variable : year				Number of groups	=	140
Number of instruments = 113				Obs per group: min =		6
Wald chi2(12) = 4883.35				avg =		6.36
Prob > chi2 = 0.000				max =		8

		Robust				
		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
n						
L1.		.8709811	.0441784	19.72	0.000	.784393 .9575692
w						
--.		-.78236	.1166924	-6.70	0.000	-1.011073 -.5536472
L1.		.512276	.1685898	3.04	0.002	.1818462 .8427059
k						
--.		.472313	.0703289	6.72	0.000	.3344709 .6101551
L1.		-.3595169	.0716555	-5.02	0.000	-.4999591 -.2190748
tau1978		.0052094	.0207006	0.25	0.801	-.035363 .0457818
tau1979		.0181131	.0242935	0.75	0.456	-.0295012 .0657275
tau1980		.0027193	.0247796	0.11	0.913	-.0458478 .0512864
tau1981		-.0208888	.0296276	-0.71	0.481	-.0789579 .0371803
tau1982		.0155336	.0272278	0.57	0.568	-.0378319 .0688991
tau1983		.0314346	.0253854	1.24	0.216	-.0183198 .081189
tau1984		.0203841	.0314453	0.65	0.517	-.0412474 .0820157
_cons		1.00307	.3916881	2.56	0.010	.2353753 1.770765

Hansen test of overid. restrictions: chi2(106) = 111.96 Prob > chi2 = 0.327

Arellano-Bond test for AR(1) in first differences: z = -5.99 Pr > z = 0.000
 Arellano-Bond test for AR(2) in first differences: z = -0.16 Pr > z = 0.872

. xtabond2 n l.n l(0/1).(w k) tau1978-tau1984, iv(tau1978-tau1984, eq(both)) gmm(n w k, lag(2
 .) eq(both)) h(3) robust

Favoring speed over space. To switch, type or click on mata: mata set matafavor space.

Arellano-Bond dynamic panel-data estimation, one-step system GMM results

Group variable: id				Number of obs	=	891
Time variable : year				Number of groups	=	140
Number of instruments = 113				Obs per group: min =		6
Wald chi2(12) = 13267.69				avg =		6.36
Prob > chi2 = 0.000				max =		8

		Robust				
		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
n						
L1.		.9326197	.0265224	35.16	0.000	.8806368 .9846025

w						
--.	-.6305321	.1195813	-5.27	0.000	-.8649071	-.396157
L1.	.4597504	.1458203	3.15	0.002	.1739479	.745553
k						
--.	.4820807	.0538588	8.95	0.000	.3765195	.5876419
L1.	-.4203043	.0587666	-7.15	0.000	-.5354847	-.3051239
tau1978	.004515	.0194143	0.23	0.816	-.0335364	.0425664
tau1979	.0186472	.0225926	0.83	0.409	-.0256335	.0629279
tau1980	.0038579	.0223114	0.17	0.863	-.0398716	.0475874
tau1981	-.0224858	.0277068	-0.81	0.417	-.0767901	.0318185
tau1982	.0117896	.027819	0.42	0.672	-.0427347	.0663139
tau1983	.0258224	.025958	0.99	0.320	-.0250543	.0766991
tau1984	.020294	.0300096	0.68	0.499	-.0385237	.0791117
_cons	.6048596	.2317178	2.61	0.009	.150701	1.059018

Hansen test of overid. restrictions: chi2(106) = 109.87 Prob > chi2 = 0.379

Arellano-Bond test for AR(1) in first differences: z = -5.37 Pr > z = 0.000
 Arellano-Bond test for AR(2) in first differences: z = -0.27 Pr > z = 0.786

Considerations.

- Ther model is treated as a system of equations, one for each time period, where now: the predetermined and endogenous variables in first-differences are instrumented with suitable lags of their own levels; the predetermined and endogenous variables in levels are instrumented with suitable lags of their own first differences; strictly exogenous regressors, as well as any other instruments, enter the instrument matrix in the conventional instrumental variables fashion: with one column per instrument.
- The default of xtabond2 in estimating GMM-sys is h (3). First-difference equations are treated as being for periods 1 to T and levels equations as being for periods T+1 to 2T. Observations for the levels equations hold the original values of the variables, while those for the first-differenced equations are of course first-differenced. The blocks of H are then 2Tx2T, and are a priori estimates of the VCOV matrix of the vector $(\Delta v' v')'$; however, in the system GMM case, there is not as natural a choice for it. As a result, several variants have been used. In general, still assuming that $E(\epsilon\epsilon')$ is the identity matrix, i.e. ϵ is homoskedastic, the VCOV matrix of the vector $(\Delta v' v')'$ will have blocks like:

$$\begin{array}{ccccccc}
 2 & -1 & 0 & \dots & 1 & 0 & 0 & \dots \\
 -1 & 2 & -1 & \dots & -1 & 1 & 0 & \dots \\
 0 & -1 & 2 & \dots & 0 & -1 & 1 & \dots \\
 \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots \\
 1 & -1 & 0 & \dots & 1+s1 & 0 & 0 & \dots \\
 0 & 1 & -1 & \dots & 0 & 1+s2 & 0 & \dots \\
 0 & 0 & 1 & \dots & 0 & 0 & 1+s3 & \dots \\
 \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots
 \end{array}$$

where $s_i = \text{Var}(\mu_i)$. For example, a diagonal element in the bottom right quadrant is: $\text{Var}(\mu_i + \epsilon_{it}) = \text{Var}(\mu_i) + 2\text{Cov}(\mu_i, \epsilon_{it}) + \text{Var}(\epsilon_{it}) = \text{Var}(\mu_i) + 2*0+1 = 1+s_i$. Similarly, on the diagonals of the bottom left and upper right quadrants, we have: $\text{Cov}(v_{it}, \Delta v_{it}) = \text{Cov}(v_{it}, \Delta \epsilon_{it}) = \text{Cov}(\mu_i, \Delta \epsilon_{it}) + \text{Var}(\epsilon_{it}) - \text{Cov}(\epsilon_{it}, \epsilon_{it-1}) = 0+1-0=1$.

- Blundell-Bond use, in their 1998 paper, h (1). The matrix h (2) is the one used in the 2002 version of DPD for Ox. Note how results change according to the choice of the a priori estimate of the covariance matrix of the error terms.