STOCHASTIC FRONTIER MODELS

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1 Introduction

Stochastic frontier models allow to analyse technical inefficiency in the framework of production functions. Production units (firms, regions, countries, etc.) are assumed to produce according to a common technology, and reach the frontier when they produce the maximum possible output for a given set of inputs. Inefficiencies can be due to structural problems or market imperfections and other factors which cause countries to produce below their maximum attainable output.

Over time, production units can become less inefficient and catch up to the frontier.[1] It is also possible that the frontier shifts, indicating technical progress. In addition, production units can move along the frontier by changing input quantities. Finally, there can be some combinations of these three effects. The stochastic frontier method allows to decompose growth into changes in input use, changes in technology and changes in efficiency, thus extending the widely used growth accounting method.

When dealing with productivity, two main problems arise: its definition and its measurement. Traditionally, empirical research on productivity has suffered from a number of shortcomings. Most empirical studies have employed the so called Solow residual (Solow 1956). The use of this measure is problematic: Abramovitz (1956) refers to the difference between the growth rates of output and the weighted sum of input growth rates as a "measure of our ignorance about the causes of economic growth". There are studies which associate productivity change measured by the resiudal with technical change (Solow 1956, Kendrick 1961, 1976, Maddison 1987). Other studies decompose productivity change into a term due to technical change and a

[1]For the importance of infrastructure investment for efficiency in the case of regional production functions in Italy, see Mastromarco and Woitek (2006).

term due to scale economies (Denison 1962, 1979, 1985). To distinguish the sources of productivity change, it is desirable to incorporate the possibility of changes in efficiency. The stochastic frontier method allows this important step.

Section 2 discusses other productivity measures proposed in the literature and discusses their advantages and drawbacks. In Section 3, both the deterministic and stochastic frontier approaches are introduced. Section 4 discusses in detail stochastic frontier analysis for cross-section models. Section 5 extends the discussion to panel data models, distinguishing the case of time invariant inefficiency from the case where inefficiency changes over time. Section 6 describes Battese and Coelli's (1995) model.

2 Growth Accounting and the Solow Residual

In empirical research, technological change has been measured as change in total factor productivity (TFP) in the analytical framework of a production function. The usual measure for technological progress is a residual of the Abramovitz/Solow type where output growth is decomposed into a weighted sum of input growth rates. The residual representing the change in output which cannot be explained by input growth is identified as technological progress.

Consider a two-factor production frontier with Hicks-neutral technical progress:

$$Y = \Theta F(L, K) \tag{1}$$

where Y is real output; Θ stands for an index of Hicks neutral technical progress; L and K are labour and capital inputs.

Taking logs on both sides of equation (1) and differentiating with respect to time yields:

$$\frac{\dot{Y}}{Y} = \frac{\dot{\Theta}}{\Theta} + \frac{\Theta F_L L}{Y}\frac{\dot{L}}{L} + \frac{\Theta F_K K}{Y}\frac{\dot{K}}{K} \tag{2}$$

where $\frac{\Theta F_L L}{Y} = e_L$ and $\frac{\Theta F_K K}{Y} = e_K$ are elasticities of output with respect to labour and capital, and $e = e_L + e_K$.

The Solow residual ($\frac{\dot{\Theta}}{\Theta}$) is given as the difference in the growth of output and the contribution of the inputs weighted by their respective factor shares in value added:

$$\frac{\dot{\Theta}}{\Theta} = \frac{\dot{Y}}{Y} - \frac{\Theta F_L L}{Y}\frac{\dot{L}}{L} - \frac{\Theta F_K K}{Y}\frac{\dot{K}}{K} \tag{3}$$

In principle this equation could be used to derive total factor productivity growth ($\frac{\dot{\Theta}}{\Theta}$) but the marginal products of labour ($\frac{\Theta F_L L}{Y}$) and capital ($\frac{\Theta F_K K}{Y}$) are not observable. This problem can be overcome by solving the firm's cost minimisation problem:

$$\min_{L,K} C = wL + rK \tag{4}$$

$$s.t.\ Y = \Theta F(L,K) \tag{5}$$

where w is wage rate and r is interest rate.

The Lagrangian is given by:

$$L(L,K,\lambda) = wL + rK + \lambda(Y - \theta F(L,K)) \tag{6}$$

The first order conditions are:

$$\Theta F_L = \frac{w}{\lambda}; \Theta F_K = \frac{r}{\lambda} \tag{7}$$

where the multiplier λ can be interpreted as marginal cost. If perfect competition is assumed, λ can be replaced by the observable market price of output P. Hence the first order condition can be rewritten as:

$$\Theta F_L = \frac{w}{P}; \Theta F_K = \frac{r}{P} \tag{4'}$$

Substituting this equation into equation (2) we obtain:

$$\frac{\dot{Y}}{Y} = \frac{\dot{\Theta}}{\Theta} + \frac{wL\dot{L}}{PYL} + \frac{rK\dot{K}}{PYK} \tag{8}$$

Therefore, we can express total factor productivity growth as the difference between output growth and weighted input growth, with revenue shares as weights:

$$\frac{\dot{\Theta}}{\Theta} = \frac{\dot{Y}}{Y} - \frac{wL\dot{L}}{PYL} - \frac{rK\dot{K}}{PYK} \tag{9}$$

This measure is, however, subject to criticism. The Solow residual ignores monopolistic markets, non-constant returns to scale and variable factor utilisation over the cycle (Saint-Paul 1997). In the case of monopoly profits, the residual underestimates the elasticity of output with respect to all inputs. To overcome this problem, Hall (1990) uses cost based shares in the derivation of his alternative TFP measure. Basu (1996) provides a measure of TFP which is net of cyclical factor utilisation. Material inputs do not have a utilisation dimension, unlike employment and capital. Basu therefore uses relative changes in the input of raw materials and other measured factor inputs to deduce the extent to which factor utilisation changes over the cycle. Another approach is the one proposed by Basu and Kimball (1997) and Basu and Fernald (2000). They link unobservable factor utilisation (U, E) to observable inputs (H) and arrive at the decomposition

$$dy = \gamma\left(s_K dk + s_L(dh + dl)\right) + \gamma\left(s_K\frac{\eta}{\nu} + s_L\zeta\right)dh + dz, \tag{10}$$

where ζ is the steady-state elasticity of hourly effort with respect to hours, η is the rate of change of the elasticity of labor costs with respect to hours, and ν is the rate of change of the elasticity of labor costs with respect to capital utilisation.[2] This decomposition can be estimated, provided that data is available.[3] The availability issue makes it necessary to apply other, less data intensive methods.

Empirical studies based on the (uncorrected) Solow residual described above regard productivity growth and technical progress as synonymous (Jorgenson 1996, Crafts 2004). However, technical progress is the change in the best practice frontier, i.e. a shift of the production function. Other productivity changes, as learning by doing, improved managerial practice, diffusion of new technological knowledge, and short run adjustment to external shocks are technical efficiency changes (movements towards or away from the frontier). Productivity growth is the net change in output due to changes in efficiency and technical change. Therefore, efficiency is a component of productivity.[4] To fix ideas, consider the example in Figure 1. It compares the output of two production units, A and B, as a function of labour, L. Given the same production technology, the higher output in country A than B can

[2]The assumption is that unobservable labour effort and capital utilization depend on observable worked hours.

[3]See Malley et al. (forthcoming) for an application to the US manufacturing sector.

[4]Nishimizu and Page (1982), Grosskopf (1993).

occur for four possible reasons. First, this difference can be due to differences in input levels, as is the case in panel (I). Second, technology acquisition may differ between production units or regions, with the consequence that for the same level of inputs different outputs result (panel (II)). Third, it might be that country B produces less efficiently than country A. In other words, both production units have the same frontier and the same input level, but output in B is lower (panel (III)). And fourth, differences could be due to some combination of the three causes. The Solow residual fails to discriminate between the second and the third possibility: efficiency is part of the residual.

As pointed out above, corrections to the Solow residual like the one proposed by Basu (1996) require data which are not always available. An additional drawback of the growth accounting approach is that the mechanical decomposition of output growth rates does not provide a direct, model based explanation of growth differences across production units.[5] Cross-country growth regressions of the Barro-type (Barro 1999) try to overcome this problem by assuming a linear relationship between several conditioning variables and growth. However, this approach is not immune against criticism: the choice of explanatory variables might be arbitrary, and the error term has no structure.[6] Thus, as in the case of the Solow residual, it is not possible to identify efficiency changes.

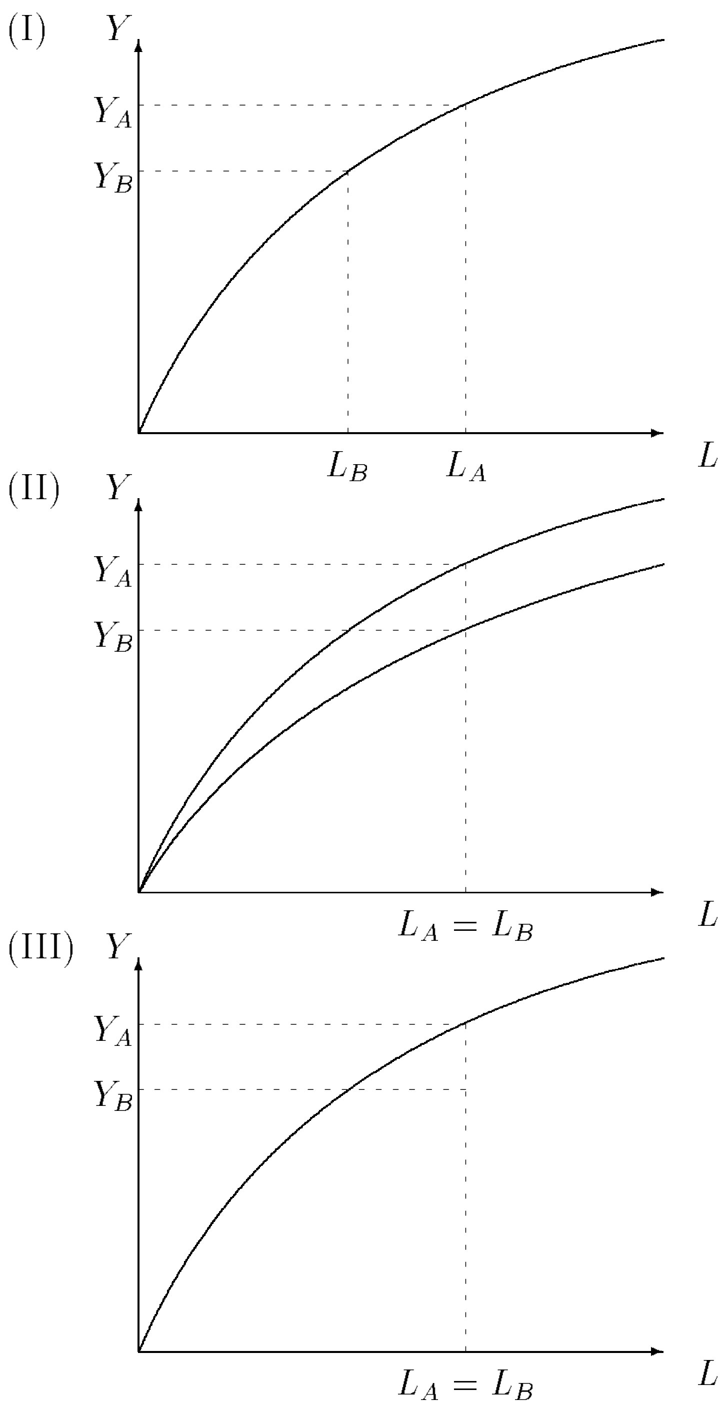
Another less data intensive approach is the estimation of a frontier production function. The stochastic frontier methodology, pioneered by Aigner et al. (1977) and Meeusen and van den Broeck (1977), allows the important distinction between efficiency gains or losses and technical progress. In

[5] Of course, after the decomposition, one could regress e.g. the residual on explanatory variables, which is a problematic approach (Wang and Schmidt 2002).

[6] See Temple (1999) and the introduction for a more detailed discussion.

addition, it allows to include explanatory variables in both the production function and the efficiency term.

Figure 1: Production Functions



3 The Production Frontier

The standard definition of a production function is that it gives the maximum possible output for a given set of inputs, the production function therefore defines a boundary or a frontier. All the production units on the frontier will be fully efficient. Efficiency can be of two kinds: technical and allocative. Technical efficiency is defined either as producing the maximum level of output given inputs or as using the minimum level of inputs given output. Allocative efficiency occurs when the marginal rate of substitution between any of the inputs equals the corresponding input price ratio. If this equality is not satisfied, it means that the country is not using its inputs in the optimal proportions. An initial justification for computing efficiency can be found in that its measure facilitates comparisons across economic units. Secondly, and perhaps more importantly, when divergence in efficiency is found some further research needs to be undertaken to understand which factors led to it. Finally, differences in efficiency show that there is scope for implementing policies addressed to reduce them and to improve efficiency.

Technical efficiency can be modelled using either the deterministic or the stochastic production frontier. In the case of the deterministic frontier model the entire shortfall of observed output from maximum feasible output is attributed to technical inefficiency, whereas the stochastic frontier model includes the effect of random shocks to the production frontier. There are two alternative approaches to estimate frontier models: one is a non-parametric approach which uses linear programming techniques, the other is a parametric approach and utilises econometric estimation. The characterising feature and main advantage of the non-parametric approach, (also called "Data Envelopment Analysis", or DEA), is that no explicit functional form needs to be imposed on the data. However, one problem with this approach is that it is extremely sensitive to outlying observations (Aigner and Chu 1968, Timmer 1971). Therefore, measures of production frontiers can produce misleading information. Moreover, standard DEA produces efficiency "measures" which are point estimates: there is no scope for statistical inference and therefore it is not possible to construct standard errors and confidence intervals.

The parametric or statistical approach imposes a specification on the production function which of course can be overly restrictive. This approach does, however, have the advantage of allowing for statistical inference. Hence, we can test the specification as well as different hypotheses on the efficiency term and on all the other estimated parameters of the production frontier. The choice of technique employed to obtain estimates of the parameters describing the structure of the production frontier and technical efficiency depends, in part, on data availability. The main difference between cross-sectional and panel-data estimation techniques is that with cross-sectional data it is only possible to estimate the performance of each producer at a specific period in time, whereas with panel data, we are able to estimate the time pattern of performance for each producer.[7] One problem with cross cross sectional data in efficiency measurement is that technical inefficiency cannot be separated from firm specific effects that are not related to inefficiency (Battese and Coelli 1995). Panel data avoids this problem.[8] Panel data contains more information than a single cross section, it therefore enables to relax some strong assumptions used in cross-sectional data and to

[7]It is assumed that producers produce only a single output. In the case of multiple outputs, these are aggregated it into a single-output index. Kumbhakar and Lovell (2000, pp. 93-95) discuss the analysis of stochastic distance functions which accommodate for multiple outputs.

[8]While implementing inefficiency measurement using panel data, it is important to distinguish technical inefficiency from firm and time specific effects. These effects are normally separate from exogenous technical progress. In a panel data context, it is possible to decompose the error into firm specific effects, time specific effects, the white noise and technical inefficiency (Kumbhakar 1991).

obtain estimates of technical efficiency with more desirable statistical properties.

A production frontier model can be written as:

$$y_i = f(\mathbf{x}_i;\boldsymbol{\beta})TE_i \tag{11}$$

where y_i is the output of producer i ($i= 1, \ldots, N$); $\mathbf{x}_i$ is a vector of M inputs used by producer i; $f(\mathbf{x}_i;\boldsymbol{\beta})$ is the production frontier and $\boldsymbol{\beta}$ is a vector of technology parameters to be estimated. Let TE_i be the technical efficiency of producer i,

$$TE_i = \frac{y_i}{f(\mathbf{x}_i;\boldsymbol{\beta})}, \tag{12}$$

which defines technical efficiency as the ratio of observed output y_i to maximum feasible output $f(\mathbf{x}_i;\boldsymbol{\beta})$. In the case $TE_i = 1$, y_i achieves its maximum feasible output of $f(\mathbf{x}_i;\boldsymbol{\beta})$. If $TE_i < 1$, it measures technical inefficiency in the sense that observed output is below the maximum feasible output. The production frontier $f(\mathbf{x}_i;\boldsymbol{\beta})$ is deterministic. That means that the entire shortfall of observed output y_i from maximum feasible output $f(\mathbf{x}_i;\boldsymbol{\beta})$ is attributed to technical inefficiency. Such a specification ignores the producer-specific random shocks that are not under the control of the producer. To incorporate the fact that output can be affected by random shocks into the analysis, we have to specify the stochastic production frontier

$$y_i = f(\mathbf{x}_i;\boldsymbol{\beta}) \exp{(v_i)}\, TE_i, \tag{11$'$}$$

where $f(\mathbf{x}_i;\boldsymbol{\beta}) \exp{(v_i)}$ is the stochastic frontier, which consists of a deterministic part $f(\mathbf{x}_i;\boldsymbol{\beta})$ common to all producers and a producer-specific part $\exp{(v_i)}$ which captures the effect of the random shocks to each producer. If we specify that the production frontier is stochastic, equation (12) becomes

$$TE_i = \frac{y_i}{f(\mathbf{x}_i;\boldsymbol{\beta})\exp(v_i)}. \tag{12$'$}$$

If $TE_i = 1$, producer i achieves its maximum feasible value of $f(\mathbf{x}_i;\boldsymbol{\beta})\exp(v_i)$. If $TE_i < 1$, it measures technical efficiency with random shocks $\exp(v_i)$ incorporated. These shocks are allowed to vary across producers.

Technical efficiency can be estimated using either the deterministic production frontier model given by equations (11) and (12), or the stochastic frontier model given by equations (11$'$) and (12$'$). Since the stochastic frontier model includes the effect of random shocks on the production process, this model is preferred to the deterministic frontier.

4 Cross-Section Stochastic Frontier Models

4.1 Introduction

The econometric approach to estimate frontier models uses a parametric representation of technology along with a two-part composed error term. Under the assumption that $f(\mathbf{x}_i;\boldsymbol{\beta})$ is of Cobb-Douglas type, the stochastic frontier model in equation (11$'$) can be written in logs as

$$y_i = \alpha + \mathbf{x}_i\boldsymbol{\beta} + \varepsilon_i \qquad i = 1, \ldots, N, \tag{11$''$}$$

where ε_i is an error term with

$$\varepsilon_i = v_i - u_i. \tag{13}$$

The economic logic behind this specification is that the production process is subject to two economically distinguishable random disturbances: statistical noise represented by v_i and technical inefficiency represented by u_i. There are some assumptions necessary on the characteristics of these components. The errors v_i are assumed to have a symmetric distribution, in particular, they are independently and identically distributed as $N\left(0, \sigma_v^2\right)$. The component u_i is assumed to be distributed independently of v_i and to satisfy $u_i \geq 0$ (e.g. it follows a one-sided normal distribution $N^+\left(0, \sigma_u^2\right)$. The non-negativity of the technical inefficiency term reflects the fact that if $u_i > 0$ the country will not produce at the maximum attainable level. Any deviation below the frontier is the result of factors partly under the production units's control, but the frontier itself can randomly vary across firms, or over time for the same production unit. This last consideration allows the assertion that the frontier is stochastic, with a random disturbance v_i being positive or negative depending on favourable or unfavourable external events.

It is important to note that given the non-negativity assumption on the efficiency term, its distribution is non-normal and therefore the total error term is asymmetric and non-normal. This implies that the least squares estimator is inefficient. Assuming that v_i and u_i are distributed independently of $\mathbf{x}_i$, estimation of (11″) by OLS provides consistent estimators of all parameters but the intercept, since $E(\varepsilon_i) = -E(u_i) \leq 0$. Moreover, OLS does not provide an estimate of producer-specific technical efficiency. However, it can be used to perform a simple test based on the skewness of empirical distribution of the estimated residuals. Schmidt and Lin (1984) propose the test statistic

$$(b_1)^{1/2} = \frac{m_3}{{m_2}^{3/2}} \tag{14}$$

where m_2 and m_3 are the second and the third moments of the empirical distribution of the residuals. Since v_i is symmetrically distributed, m_3 is simply the third moment of the distribution of u_i.

The case $m_3 < 0$ implies that OLS residuals are negatively skewed, and that there is evidence of technical inefficiency. In fact, if $u_i > 0$ then $\varepsilon_i = v_i - u_i$ is negatively skewed. The positive skewness in the OLS residuals, i.e. $m_3 > 0$, suggests that the model is misspecified. Coelli (1995) proposed an alternative test statistic

$$(b_1)^{1/2} = \frac{m_3}{(6m_2^3/N)^{1/2}}, \tag{15}$$

where N is equal to the number of observations. Under the null hypothesis of zero skewness in the OLS residuals, $m_3 = 0$, the third moment of OLS residuals is asymptotically distributed as a normal random variable with mean zero and variance $\frac{6m_2^3}{N}$. This implies that the test statistic $(b_1)^{1/2} = m_3/(6m_2^3/N)^{1/2}$ (eq. 15) is asymptotically distributed as a standard normal random variable $N(0,1)$.

These two tests have the advantage that they can easily be computed given that they are based on the OLS residuals. They have the disadvantage that they rely on asymptotic theory and therefore are not suitable for small samples. Coelli (1995) presents Monte Carlo experiments where these tests have the correct size and good power.

The asymmetry of the distribution of the error term is a central feature of the model. The degree of asymmetry can be represented by the following parameter:

$$\lambda = \frac{\sigma_u}{\sigma_v} \quad . \tag{16}$$

The larger λ is, the more pronounced the asymmetry will be. On the other hand, if λ is equal to zero, then the symmetric error component dominates the one-side error component in the determination of ε_i. Therefore, the complete error term is explained by the random disturbance v_i, which follows a normal distribution. ε_i therefore has a normal distribution. To test the hypothesis that $\lambda = 0$, we can compute a Wald statistic or likelihood ratio test both based on the maximum likelihood estimator of λ.[9] Coelli (1995) tests as equivalent hypothesis $\gamma = 0$ against the alternative $\gamma > 0$, where

$$\gamma = \frac{\sigma_u}{\sigma_v + \sigma_u}. \tag{17}$$

A value of zero for the parameter γ indicates that the deviations from the frontier are entirely due to noise, while a value of one would indicate that all deviations are due to technical inefficiency.[10]

The Wald statistic is calculated as

$$W = \frac{\widehat{\gamma}}{\widehat{\sigma}_{\widehat{\gamma}}}, \tag{18}$$

where $\widehat{\gamma}$ is maximum likelihood estimate of γ and $\widehat{\sigma}_{\widehat{\gamma}}$ is its estimated standard error. Under $H_0 : \gamma = 0$ is true, the test statistic is asymptotically distributed as a standard normal random variable. However, given that γ cannot be negative, the test is performed as a one-sided test. The likelihood test statistic is

$$LR = -2\left[\log\left(L_0\right) - \log\left(L_1\right)\right], \tag{19}$$

where $\log\left(L_0\right)$ is the log-likelihood valued under the null hypothesis and

[9] Coelli (1995) shows that the likelihood ratio test is asymptotically distributed as a mixture of Chi squared distributions.

[10] Coelli (1995) stresses that the parameter does not reflect the contribution of the inefficiency effect to the total variance, since the variance of inefficiency is not equal to σ_u^2 but to $\left[(\pi-2)/\pi\right]\sigma_u^2$. Therefore, the contribution of the inefficiency effect to the total variance is equal to $\gamma/[\gamma + (1-\gamma)\,\pi/(\pi-2)]$.

$\log(L_1)$ is the log-likelihood value under the alternative. This test statistic is asymptotically distributed as chi-square random variable with degrees of freedom equal to the number of restrictions.[11] Coelli (1995) notes that under the null hypothesis $\gamma = 0$, the statistic lies on the limit of the parameter space since γ cannot be less than zero.[12] He therefore concludes that the likelihood ratio statistic will have an asymptotic distribution equal to a mixture of chi square distributions $\left(1/2\right)\chi_0^2 + \left(1/2\right)\chi_1^2$. Kodde and Palm (1986) present critical values for this test statistic. Coelli (1995), performing a Monte Carlo study, shows that the Wald test has very poor size. With a confidence interval of 5%, the Wald test rejects the null hypothesis 20% times instead of 5% as expected (Type I error). The likelihood ratio test instead has the correct size and superior power with respect to the Wald test and the test based on the third moment of the OLS residuals. Coelli concludes that this test should be performed with maximum likelihood estimation.

Conventionally, the efficiency term can take the form of a truncated normal distribution, of a half-normal distribution, of an exponential distribution, or of a gamma distribution. The density function in the truncated normal case is defined by

$$f(u_i) = \frac{\exp\left[-\frac{1}{2}(u_i - \mu)^2/\sigma_u^2\right]}{(2\pi)^{1/2}\sigma_u\left[\Phi(-\mu/\sigma_u)\right]}, \qquad u_i > 0, \tag{20}$$

where $\Phi(.)$ is the cumulative distribution function (cdf) of the standard normal random variable. If a half-normal distribution for the inefficiency component is assumed, equation (20) can be modified simply by imposing a zero

[11] In this case, the number of restrictions is equal to one.

[12] Because this would mean a negative variance of the inefficiency term σ_u^2.

mean, i.e. $\mu = 0$. Therefore, the density function of the term u becomes

$$f(u_i) = 2\frac{\exp\left[-\frac{1}{2}(u_i)^2/\sigma_u^2\right]}{(2\pi)^{1/2}\sigma_u}, \qquad u_i > 0. \tag{21}$$

This can be explained by the fact that the normal distribution function evaluated at zero is one half.[13]

In the exponential case, the distribution function of the inefficiency term will take the form

$$f(u_i) = \rho^{-1}\exp(-\rho^{-1}\ u_i), \qquad u_i > 0, \tag{22}$$

where ρ is the parameter of the exponential distribution to be estimated. The inverse of ρ is equal to the mean of the distribution itself, that is $E\left(u_i\right) = \frac{1}{\rho}$ and the variance $\sigma_u^2 = \frac{1}{\rho^2}$.[14] Finally, in the case where efficiency follows a gamma distribution, the density function will be equal to

$$f(u_i) = \frac{u_i^m}{\Gamma\left(m+1\right)\sigma_u^{m+1}}\exp\left(-\frac{u_i}{\sigma_u}\right), \qquad u_i > 0. \tag{23}$$

The gamma distribution is a two-parameter distribution, depending on m and σ_u. If $m = 0$, the gamma density function becomes the density function of the exponential distribution.

4.2 Problems related to the Estimation of the Model

It has been demonstrated here that to estimate a stochastic frontier model, several strong assumptions need to be imposed, in particular about the distribution of statistical noise (normal) and of technical inefficiency (e.g. one-

[13]When $\mu = 0$, $\Phi(-\mu/\sigma) = \Phi(0) = \frac{1}{2}$.

[14]Thus, given that $\sigma_u^2 = \frac{1}{\rho^2}$ and $E\left(u_i\right) = \frac{1}{\rho}$, the final expression when the efficiency follows an exponential distribution is: $f(u_i) = \sigma_u^{-1}\exp(-\sigma_u^{-1}\ u_i)$.

sided normal). In addition, the assumption that inefficiency is independent of the regressor may be incorrect, because, as argued by Schmidt and Sickles (1984), "if a firm knows its level of technical inefficiency, this should affect its input choices". These problems can be solved by the use of panel data (Section 5). Early panel data studies hypothesised that the intercept and the inefficiency component of the error term are time-invariant, so that the country effect $\alpha_i = \alpha - u_i$ could be estimated without distributional assumptions and then be converted into measures of inefficiency. This time-invariance assumption therefore makes it possible to substitute for many of the strong assumptions necessary in the case of a single cross-section. Recent panel data literature has tried to relax the assumption of a time-invariant inefficiency component (Cornwell and Schmidt 1996).

4.3 Estimation Methods

There are two main methods to estimate the stochastic frontier models: one is the Modified Ordinary Least Squares (MOLS) methodology, the other consists of maximising the likelihood function directly. The following two sections present an overview of each methodology.

4.4 Modified Ordinary Least Squares (MOLS)

For the system in equations ($11''$) and (13) all the assumptions of the classical regression model apply, with the exception of the zero mean of the disturbances ε_i. The OLS estimator will be a best linear unbiased and consistent estimate of the vector β. Problems arise for the intercept term α: its OLS estimate is not consistent. To illustrate this, a simple model where there is only the intercept, i.e. $y_i = \alpha + \varepsilon_i$ can be considered. The OLS estimator of the parameter α would be the mean of y, $\bar{y}$, which has plim $\bar{y} = \alpha + \mu_\varepsilon \neq \alpha$. The bias of the constant term is given by the mean of the error term μ_ε.

Winsten (1957) proposes corrected ordinary least squares (COLS) to estimate the production frontier. In the first step Ordinary Least Squares (OLS) is used to obtain consistent and unbiased estimates of the slope parameters and a consistent but biased estimate of the intercept. In the second step, the estimated intercept is shifted up by the maximum value of the OLS residuals. The COLS intercept is estimated consistently by $\alpha + \max_i \hat{u}_i$, where $\hat{u}_i$ is the OLS residual at observation i. The OLS residuals are corrected in the opposite direction: $-\hat{u}'_i = \hat{u}_i - \max_i \hat{u}_i$.

Afriat (1972) and Richmond (1974) propose the MOLS procedure.[15] The MOLS technique consists of correcting the intercept with the expected value of the error term[16] (ε_i) and adopting OLS to get a consistent estimate. In the case of the half normal distribution, the mean of ε_i given by

$$\mu_\varepsilon = \sigma_u \sqrt{2/\pi}, \tag{24}$$

where σ_u is the standard deviation of the inefficiency term. The OLS intercept estimator is consistent for $\alpha + \mu_\varepsilon$, where σ_u has been substituted by its estimate $\hat{\sigma}_u$:

$$\hat{\sigma}_u^2 = \left[\sqrt{\pi/2}\left(\frac{\pi}{\pi - 4}\right)\hat{m}_3\right]^{2/3} \qquad \text{and} \qquad \widehat{\sigma}_v^2 = \hat{m}_2 - \left(1 - \frac{2}{\pi}\right)\widehat{\sigma}_u^2. \tag{25}$$

[15] This procedure is very similar to the two-step COLS procedure.

[16] Afriat (1972) and Richmond (1974) explicitly assume that the disturbances follow a one-sided distribution, such as exponential or half normal.

The parameters $\hat{m}_3$ and $\hat{m}_2$ are the third and second moments of the OLS residuals.[17] To summarise, the estimate of σ_u is used to convert the OLS estimate of the constant term into the MOLS estimate. The model to be estimated is

$$y_i = (\alpha + \mu_\varepsilon) + \boldsymbol{\beta}\mathbf{x}_i + \varepsilon_i. \tag{26}$$

As in COLS model, the OLS residuals provide consistent estimates of the technical efficiency of each unit $-\hat{u}_i' = \hat{u}_i - \mu_\varepsilon$.[18] The estimation by OLS will lead to consistent but inefficient estimates of all the parameters. A problem with the MOLS technique is that the estimates can take values which have no statistical meaning. Suppose the third moment of the OLS residuals is positive, then the term in brackets in equation (25) becomes negative and this leads to a negative value of $\hat{\sigma}_u$. Olson et al. (1980) label this failure as a Type I Error. A Type II Error occurs when $\widehat{\sigma}_\varepsilon^2 < \left[\left(\pi - {}^2\!/_\pi\right)\widehat{\sigma}_u^2\right]$ and implies that $\widehat{\sigma}_v^2 < 0$.

Moreover, the estimated production frontier is parallel to the OLS regression, since only the OLS intercept is corrected.[19] This implies that the structure of the "best practice" production technology is the same as the structure of the "central tendency" production technology. This is an un-

[17]The error term is $\epsilon_i = v_i - u_i$. In the case $v_i \sim N\left(0,\, \sigma_v^2\right)$ and u_i follows a half normal distribution, the first, second and third moments of the efficiency term are: $E\left(u_i\right) = \sqrt{{}^2\!/_\pi}$, $E\left(u_i^2\right) = \left[\left(\left(\pi - 2\right)\!/_\pi\right)\right]\sigma_u^2$ and $E\left(u_i^3\right) = \left[-\sqrt{{}^2\!/_\pi}\left(1 - {}^4\!/_\pi\right)\right]\sigma_u^3$. This implies that the second and the third central moments of ε_i are: $E\left(\varepsilon_i^2\right) = \sigma_v^2 + \left[\left(\pi - 2\right)\!/_\pi\right]\sigma_u^2$ and $E\left(\varepsilon_i^3\right) = \left[\sqrt{{}^2\!/_\pi}\left(1 - {}^4\!/_\pi\right)\right]\sigma_u^3$. Then the second (m_2) and third moments (m_3) of the OLS residuals are used to estimate σ_u^2 and σ_v^2 (equation 25).

[18]MOLS procedure does not ensure that all units are bounded from above by the estimated production frontier. If a unit has a large positive OLS residual then it is possible that $u_i - \mu_\varepsilon > 0$; thus technical efficiency score is greater than unit. This result is difficult to explain and represents a drawback of this method.

[19]This problem also affects the COLS methodology.

desirably restrictive property of the MOLS procedure, since the structure of "best practice" technology ought to differ from the production technology of the producers down in the middle of the data who are less efficient than the "best practice" producer.

4.5 Maximum Likelihood Estimation

As demonstrated in the previous section, consistent estimates of all the parameters of the frontier function can be obtained simply using a modification of the OLS estimator. However the distribution of the composed error term is asymmetric (because of the asymmetric distribution of the inefficiency term). A maximum likelihood estimator that takes into consideration this information should therefore give more efficient estimates, at least asymptotically.[20] This has been investigated by Greene (1980a,b) who argues that the Gamma distribution is one of the distributions which provides a maximum likelihood estimator with all of the usual desirable properties and which is characterised by a high degree of flexibility. This distribution should therefore be used to model the inefficiency error term. However, it has been noticed that the flexibility of the Gamma distribution can make the shapes of statistical noise and inefficiency hardly distinguishable.[21] The log-likelihood function for the

[20] Koop et al. (1999, 2000a,b), and Koop (2001) adopt a Bayesian approach to estimate stochastic production frontiers. While there are certainly advantages of the Bayesian estimation method, the choice of Maximum Likelihood estimation in large sample is justified. Kim and Schmidt (2000) examine a large number of classical and Bayesian procedures to estimate the level of technical efficiency using different panel data sets. They find that Maximum Likelihood estimation based on the exponential distribution gives similar results to the Bayesian model in which the prior distribution for efficiency is exponential and there is an uninformative prior for the exponential parameter. The problem in the classical framework is that asymptotically valid inference may be not valid in small samples.

[21] See van den Broeck et al. (1994).

model defined by equations (11″) and (13) is derived by Aigner et al. (1977).[22]

When considering the half normal distribution $u_i \sim N^+(0, \sigma_u)$, the maximum log-likelihood function takes the form

$$\begin{aligned}\ln L\left(y \left| \alpha, \boldsymbol{\beta}, \lambda, \sigma^2\right.\right) =& N \ln \frac{\sqrt{2}}{\sqrt{\pi}} + N \ln \sigma^{-1} + \sum_{i=1}^{N} \ln \left[1 - \Phi(\varepsilon_i \lambda \sigma^{-1})\right] - \\ & \frac{1}{2\sigma^2} \sum_{i=1}^{N} \varepsilon_i^2,\end{aligned} \tag{27}$$

where λ is the ratio defined in equation (16), $\sigma = \sigma_u^2 + \sigma_v^2$ and $\Phi(.)$ is the standard normal cumulative distribution function.

If we assume an a truncated normal distribution $u_i \sim N^+(\mu, \sigma_u)$, the log-likelihood function is

$$\begin{aligned}\ln L\left(y \left| \alpha, \boldsymbol{\beta}, \lambda, \sigma^2\right.\right) \;=\; & -\frac{N}{2} \ln \left(\frac{\pi}{2}\right) - N \ln \sigma - N\Phi\left(\frac{-\mu}{\lambda\sigma}\right) + \\ & \sum_{j=1}^{N} \ln \Phi\left(\frac{-\mu\lambda^{-1} - \varepsilon_j \lambda}{\sigma}\right) - \frac{1}{2\sigma^2} \sum_{j=1}^{N} \varepsilon_j^2.\end{aligned} \tag{28}$$

In the case where the efficiency follows an exponential distribution $u_i \sim \mathrm{Ex}(\theta),\ \theta = \sigma_u^{-1}$, the log-likelihood function is

$$\begin{aligned}\ln L\left(y \left| \alpha, \boldsymbol{\beta}, \lambda, \sigma^2\right.\right) =& - N\left(\ln \sigma_u + \frac{\sigma_v^2}{2\sigma_u^2}\right) + \sum_{j=1}^{N} \ln \Phi\left(\frac{-\varepsilon_j}{\sigma_v} - \lambda^{-1}\right) + \\ & \sum_{j=1}^{N} \frac{\varepsilon_j}{\sigma_u}.\end{aligned} \tag{29}$$

[22] The log-likelihood function is expressed in terms of the two parameters $\sigma^2 = \sigma_u^2 + \sigma_v^2$ and $\lambda = \frac{\sigma_u}{\sigma_v}$. Given that the parameter λ can assume any non-negative value, Battese and Corra (1977) suggest to use the parameter $\gamma = \frac{\sigma_u^2}{\sigma_v^2 + \sigma_u^2}$ that can vary between zero and one. Coelli (1995) observes that $\lambda = \sqrt{\gamma/(1-\gamma)}$.

4.6 Measurement of Efficiency

Battese and Coelli (1988) define technical efficiency of a firm as the ratio of its mean production (in original units), given the level of inefficiency, to the corresponding mean production if the inefficiency level were zero. Using this definition, technical efficiency for country i, TE_i is

$$TE_i = \frac{E(y_i^* \,|u_i, \mathbf{x}_i)}{E(y_i^* \,|u_i = 0, \mathbf{x}_i)}, \tag{30}$$

where y_i^* is the value of production (in original units) for the ith country. This measure will necessarily be bound between zero and one, because the level of production under inefficiency (the economy is producing below the production frontier) will always be smaller than the level of efficient production. If it is assumed that the production function (11″) is expressed in logarithmic form, then the inefficiency term will be

$$TE_i = \exp(-u_i). \tag{31}$$

When the data are in logarithms it is notable that the measure of efficiency is equivalent to the ratio of the level of production (when inefficiency occurs), $\exp(y_i) = \exp(a + \boldsymbol{\beta}\mathbf{x}_i + v_i - u_i)$, to the corresponding value of production without inefficiency, $\exp(y_i) = \exp(a + \boldsymbol{\beta}\mathbf{x}_i + v_i)$. Because of the way technical efficiency is measured, the latter measure (31) compared to (30) is independent of the level of the inputs. The problem that now arises is how to compute this measure of efficiency. A method has been proposed by Jondrow et al. (1982), and it is based on the distribution of the inefficiency term conditional to the composite error term, $u_i|\varepsilon_i$. This distribution contains all the information that ε_i yields about u_i, therefore we can use the expected value of the distribution as a point estimate of u_i. Jondrow et al. (1982) demonstrate under the assumptions that (i) v_i are iid $N(0, \sigma_v^2)$, (ii) $\mathbf{x}_i$ and v_j are independent, (iii) u_i are independent of $\mathbf{x}_i$ and v_i, and (iv) u_i follow a one-sided normal distribution (e.g. truncated or half-normal), the distribution of $u_i|\varepsilon_i$ is a normal random variable $N(\mu_i^*, \sigma_*^2)$ where $\mu_i^* = \sigma_u^2 \varepsilon_i (\sigma_u^2 + \sigma_v^2)^{-1}$ and $\sigma_*^2 = \sigma_u^2 \sigma_v^2 (\sigma_u^2 + \sigma_v^2)^{-1}$. A point estimate for TE_i is therefore given by

$$TE_i = E\left[\exp(-u_i)\,|\varepsilon_i\right] = \frac{[1 - \Phi(\sigma_* - \mu_i^*/\sigma_*)]}{[1 - \Phi(-\mu_i^*/\sigma_*)]} \exp\left[-\mu_i^* + \frac{1}{2}\sigma_*^2\right], \qquad (32)$$

where $\Phi(.)$ is the standard normal cumulative density function. In order to implement this procedure estimates of μ_i^* and σ_*^2 are required, and therefore estimates of the variances of the inefficiency and random components and of the residuals $\hat{\varepsilon}_i = y_i - \hat{\alpha} - \mathbf{x}_i \hat{\boldsymbol{\beta}}$. Equation (32) holds when the distribution of the inefficiency component is a truncated distribution; whereas, when it follows a half-normal distribution (for which $\mu_i^* = 0$), the point estimate of technical efficiency will take the simpler form

$$TE_i = E\left[\exp(-u_i)\,|\varepsilon_i\right] = 2\left[1 - \Phi(\sigma_*)\right] \exp\left[\frac{1}{2}\sigma_*^2\right], \qquad (33)$$

where the usual notation holds.

A Monte Carlo study conducted by Kumbhakar and Löthgren (1998) shows negative bias in the estimated inefficiencies and confidence intervals to be significantly below the corresponding theoretical confidence levels.[23] The evidence is that this bias decreases as the sample size increases. Moreover, they find that the point estimator outperforms the interval estimators of technical inefficiency. Thus, the uncertainty associated with unknown pa-

[23] Kumbhakar and Löthgren (1998) assume in their Monte Carlo study that the true values of the underlying parameters are unknown and must be replaced by their ML estimates. They found that the result is true for all value of inefficiency and for sample sizes less than 200.

rameters is reduced when the number of observations increases.[24] This result supports the empirical estimations in studies where the sample size is fairly large. There are many empirical studies that show the sensitivity of the estimated efficiencies to the distribution assumption on the one-sided error component. However, Greene (1990) finds that the ranking of producers by their individual efficiency scores and the composition of the top and bottom score deciles is not sensitive to distribution assigned to the efficiency terms. Since the assumption that efficiency terms follow an half normal distribution is both plausible and tractable, it is typically employed in empirical work.[25]

5 Panel Data Stochastic Frontier Models

5.1 Introduction

In the previous sections some of the problems related to a cross-sectional analysis have been pointed out, namely the assumption that technical inefficiency is independent of the inputs and the assumptions on the distributional forms of statistical noise and technical inefficiency. Both these problems can be solved by the use of panel data. In particular, panel data allows relaxation of the assumption of independence and avoidance of distribution assumptions or testing them when they are imposed. Furthermore, with panel data it is possible to construct estimates of the efficiency levels of each country that are consistent as the number of observations per country increases. This means that inefficiency can be estimated more precisely. The general model which

[24] The Monte Carlo study is performed for sample size N=25, 50, 100, 200, 400 and 800.

[25] On this argument see Kumbhakar and Lovell (2000) pp.74-90.

will be analysed is of the form

$$y_{it} = \alpha_i + \boldsymbol{\beta}\mathbf{x}_{it} + v_{it} - u_{it} \qquad i = 1, \ldots, N; \quad t = 1, \ldots, T. \tag{34}$$

Before proceeding with the estimation of the model a distinction concerning the time dimension of the inefficiency term has to be made. In the first case the term defining inefficiency u will be kept constant over time for each country, whereas in the second case it will be allowed to change over time.

5.2 Time-Invariant Inefficiency

In this section a model with time-invariant inefficiency will be presented. Equation (34) can be rewritten as follows:

$$y_{it} = \alpha + \boldsymbol{\beta}\mathbf{x}_{it} + v_{it} - u_i \qquad i = 1, \ldots, N; \quad t = 1, \ldots, T. \tag{35}$$

By defining $\alpha_i = \alpha - u_i$ we have the standard panel data model

$$y_{it} = \alpha_i + \boldsymbol{\beta}\mathbf{x}_{it} + v_{it} \tag{36}$$

It is assumed that the v are i.i.d. $(0, \sigma_v^2)$ and uncorrelated with the inputs $\mathbf{x}$. This last assumption is needed for the consistency of the within and generalised estimators of the parameter vector $\boldsymbol{\beta}$, which are derived from the OLS estimation of equation (36) under a fixed effect model and a random effect model respectively.

5.3 Fixed Effects Model

The fixed effect model consists of treating the inefficiency levels u_i (and therefore the intercepts α_i) as fixed, as simple parameters to be estimated.

It should be noted that in this specific case, no assumptions are made on the distribution of the inefficiency term or on the correlation between the inefficiency term with the regressors and the statistical noise v_i. By applying ordinary least squares estimation to the model (36) combined for all T observations for each country, the within estimator is derived. It can be shown to be consistent as either N or T go to infinity. Once the within estimator is available, an estimate of the intercept terms α_i is possible, and therefore the country-specific technical inefficiencies can be estimated as:

$$\hat{u}_i = \hat{\alpha}^* - \hat{\alpha}_i \quad \text{where} \quad \hat{\alpha}^* = \max_i \hat{\alpha}_i. \tag{37}$$

Specification (37) means that the production frontier is normalised in terms of the best country in the sample. A necessary condition for $\hat{\alpha}_i$ to be consistent is that the time period T is very large, whereas to have an accurate normalisation and a consistent separation of α from the one-sided inefficiency terms u_i a large number of production units N is required. This means that if N is small it is only possible to compare efficiencies across production units, but not to an absolute standard (100%). In their empirical analysis on three different sets of panel data, Horrace and Schmidt (1996) find wide confidence intervals for the efficiency estimates based on the fixed-effects model. The estimation error and the uncertainty in the identification of the most efficient observation are among the explanations adopted to justify this result. A problem related to the within estimation is that if important time-invariant regressors are included in the frontier model, these will show up as inefficiency in equation (37) (Cornwell and Schmidt 1996). In other words, the fixed effects (u_i) capture both variation across producers in time-invariant technical efficiency and all phenomena that vary across producers but are time invariant for each producer. Unfortunately, this occurs whether or not the other effects are included as regressors in the model.[26] This problem can be solved by estimating model (34) in a random effect context.

5.4 Random Effects Model

In the random effects model the inefficiency terms u_i are treated as one-sided i.i.d. random variables, uncorrelated with the regressors x_{it} and the statistical noise v_{it} for all t. So far no distributional assumptions for the effects are made. Before proceeding to the estimation, the model (35) is rewritten in a slightly different way, defining $\alpha^* = \alpha - \mu$, where $\mu = E(u_i)$:

$$y_{it} = \alpha^* + \boldsymbol{\beta}\mathbf{x}_{it} + v_{it} - u_i^* \qquad where \quad u_i^* = u_i - \mu \tag{38}$$

The estimator for the random effects model is the Generalised Least Square (GLS) estimator $\left(\hat{\alpha}^* \quad \hat{\boldsymbol{\beta}}'\right)'_{GLS}$, which is consistent as N approaches infinity. The covariance matrix appearing in the estimator depends on the variances of the two components of the error term, that is σ_v^2 and σ_u^2. In the unrealistic case that these two variances are known, the GLS estimator is consistent as N goes to infinity. In the more realistic case that they are unknown, the feasible GLS (FGLS) estimator is still consistent as $N \to \infty$, if it is based on consistent estimates of σ_v^2 and σ_u^2. The advantages offered by the FGLS estimator are that it allows the inclusion of time-invariant variables and gives more efficient estimates than the within estimator of the fixed effect. Nevertheless, the efficiency advantage depends on the orthogonality of the regressors and the inefficiency term, a condition which is often rejected by the data; in addition the gain in terms of efficiency vanishes as T approaches infinity. For this reason, Schmidt and Sickles (1984) point

[26] On this argument see Kumbhakar and Lovell (2000) pp.97-100.

out that the random effects model is more suitable for short panels in which correlation is empirically rejected. Hausman and Taylor (1981) developed a test, based on Hausman (1978), for the hypothesis that the error terms are uncorrelated with the regressors. If the null hypothesis of non-correlation is accepted, a random-effects model is chosen, otherwise a fixed-effects model is appropriate. The Hausman test is a test of the orthogonality assumption that characterises the random effects estimator, which is defined as the weighted average of the between and the within estimator.[27] The test statistic is

$$H = (\hat{\boldsymbol{\beta}}_{RE} - \hat{\boldsymbol{\beta}}_{FE}) \left(\hat{\boldsymbol{\Sigma}}_{\hat{\boldsymbol{\beta}}_{FE}} - \hat{\boldsymbol{\Sigma}}_{\hat{\boldsymbol{\beta}}_{RE}} \right)^{-1} (\hat{\boldsymbol{\beta}}_{RE} - \hat{\boldsymbol{\beta}}_{FE})', \tag{39}$$

where $\hat{\boldsymbol{\beta}}_{RE}$ and $\hat{\boldsymbol{\beta}}_{FE}$ are the estimated parameter vectors from the random and the fixed effect models, and $\boldsymbol{\Sigma}_{RE}$ and $\boldsymbol{\Sigma}_{FE}$ the respective covariance matrices. Under the null hypothesis that the random effects estimator is appropriate, the test-statistic is distributed asymptotically as a χ^2 with degrees of freedom equal to the number of the regressors. Henceforth, large values of the H test-statistic have to be interpreted as supporting the fixed effects model. Hausman and Taylor (1981) developed a similar test of the hypothesis that the inefficiency terms are not correlated with the regressors. Technical inefficiency is estimated by taking the average values of FGLS residuals:

$$\hat{\varepsilon}_i = \frac{1}{T} \sum_t (y_{it} - \hat{\alpha}^* - \hat{\boldsymbol{\beta}} \mathbf{x}_{it}) \qquad where \quad \alpha^* = \alpha - \mu \tag{40}$$

and

$$\hat{u}_i = \hat{\varepsilon}_i^* - \hat{\varepsilon}_i \quad \text{where} \quad \hat{\varepsilon}_i^* = \max_i \hat{\varepsilon}_i. \tag{41}$$

The inefficicency estimates are consistent if both N and T are large

[27] See Hsiao (1986).

enough, as in the fixed effect case.

5.5 Maximum Likelihood Estimation

The main advantage in using panel data is that it allows relaxation of the strong assumptions required in the estimation of a cross-section, namely assumptions on the independence of the components of the error term and the regressors, and distributional assumptions on the inefficiency and statistical noise. Clearly, it is still possible to make these assumptions and therefore a maximum likelihood estimator of the parameters of the model can be obtained. The advantage of panel data in this context is that, as noted by Cornwell and Schmidt (1996), "repeated observation of the same firm makes it possible to estimate its level of efficiency more precisely." The Battese-Coelli estimator presented in equations (30) to (32) can therefore be generalised to the case of panel data under the same assumptions presented for the cross-section case. It is necessary to slightly modify two of the variables involved, namely μ_i^* and σ_*^2. They are the mean and the variance of the normally distributed inefficiency term conditional on the composed error term, $u_i|\varepsilon$, which appears in (32). It can now be observed that the mean and the variance of the conditional distribution are given respectively by

$$\begin{aligned} \mu_i^* &= \sigma_u^2 \bar{\varepsilon}_i (\sigma_u^2 + \sigma_v^2/T)^{-1} \\ \sigma_*^2 &= \sigma_u^2 \sigma_v^2 (\sigma_u^2 + T\sigma_v^2)^{-1}, \end{aligned} \tag{42}$$

where $\bar{\varepsilon}_i = (1/T) \sum_i \varepsilon_{it}$.

One of the advantages of using the Battese-Coelli method is that it allows for unbalanced panels, i.e. different numbers of observations per country: with T_i observations for country i, T has to be replaced by T_i in system (42).

Note that the variance will depend on i. Another advantage is that the intercept can be estimated directly, without the maximisation used in equation (37). Therefore, the best country in the sample is no longer normalised to be 100 percent efficient.

5.6 Time-Varying Inefficiency

If the assumption of a time invariant inefficiency term is relaxed, the model to be examined is the following:

$$y_{it} = \alpha_{it} + \boldsymbol{\beta}\mathbf{x}_{it} + v_{it}, \tag{43}$$

where $\alpha_{it} = \alpha_t - u_{it}$ and $u_{it} \geq 0$. Given that it is possible to estimate α_{it}, the following estimates of the inefficiency term can be obtained:

$$\hat{u}_{it} = \hat{\alpha}_t - \hat{\alpha}_{it} \qquad \text{where} \quad \hat{\alpha}_t = \max_i(\hat{\alpha}_{it}). \tag{44}$$

The problem arising here is that some restrictions are needed to estimate the intercepts α_{it}, and the aim is to find weak enough restrictions which allow for some degree of flexibility. Cornwell et al. (1990) introduced a model where the intercepts depend on a vector of observables $\mathbf{w}_t$ in the following way:

$$\alpha_{it} = \boldsymbol{\delta}_i \mathbf{w}_t = \begin{pmatrix} \delta_{i1} & \delta_{i2} & \delta_{i3} \end{pmatrix} \begin{pmatrix} 1 \\ t \\ t^2 \end{pmatrix}, \tag{45}$$

and where the effects $\boldsymbol{\delta}_i$ are fixed. As Cornwell and Schmidt (1996) point out, this specification can also "be interpreted as a model of productivity growth, with rates that differ for each firm". Country-specific productivity growth rates can be constructed as the time derivatives of equation (45). In this framework , the general model to be estimated becomes

$$y_{it} = \boldsymbol{\beta}\mathbf{x}_{it} + \boldsymbol{\delta}_i \mathbf{w}_{it} + v_{it}. \tag{46}$$

The estimation procedure starts by finding the within estimator $\hat{\boldsymbol{\beta}}_w$, then continues by applying OLS to a regression of the residuals $(y_{it} - \hat{\boldsymbol{\beta}}_w \mathbf{x}_{it})$ to find estimates of the elements of $\boldsymbol{\delta}_i$ and then computing $\hat{\alpha}_{it}$ as $\hat{\boldsymbol{\delta}}_i \mathbf{w}_{it}$ (this last estimate will be consistent for $T \rightarrow \infty$). Finally, estimates of inefficiency as in (44) will be obtained. Cornwell et al. (1990) consider the fixed-effect and the random-effects approach. Since time-invariant regressors cannot be included in the fixed-effects model, they develop a GLS random-effects estimator for time-varying technical efficiency model. However, the GLS estimator is inconsistent when the technical inefficiencies are correlated with the regressors, therefore the authors compute an efficient instrumental variables (EIV) estimator that is consistent in the case of correlation of the efficiency terms with the regressors, and that also allows for the inclusion of time-invariant regressors. Lee and Schmidt (1993) specify the term u_{it} as

$$u_{it} = \left(\sum_{t=1}^{T} \beta_t d_t \right) u_i, \tag{47}$$

where d_t is a time dummy variable and one of the coefficients is set equal to one. This formulation of technical change, differently from that of Cornwell et al. (1990), does not restrict the temporal pattern of the u_{it} apart for the β_t to be the same for all producers. This time-varying technical efficiency can be estimated with both fixed- and random-effects models, where the coefficients β_t are treated as the coefficients of u_i. Since this model requires estimation of *T-1* additional parameters, it is appropriate for short panels.[28] Once β_t and u_i are estimated, the following expression can be obtained:

$$u_{it} = \max_i \left(\hat{\beta}_t \hat{u}_i\right) - \left(\hat{\beta}_t \hat{u}_i\right), \tag{48}$$

from which the technical efficiency can be calculated as

$$TE_{it} = \exp\left(-\hat{u}_{it}\right). \tag{49}$$

If the inefficiency terms are independently distributed, maximum likelihood techniques can be used to estimate the time varying technical efficiency model. The technical efficiency adding time dummies can be specified as

$$u_{it} = \beta_t u_i. \tag{50}$$

Kumbhakar (1990) proposed the following parametric function of time for u_{it}:

$$u_{it} = u_i \left(1 + \exp\left(\delta_1 t + \delta_2 t^2\right)\right)^{-1}. \tag{51}$$

Battese and Coelli (1992) suggested an alternative specification:

$$u_{it} = u_i \left(\exp\left(-\delta(t-T)\right)\right). \tag{52}$$

Both of these models are estimated using the maximum likelihood procedure discussed in Section 5.5. Kumbhakar's 1990 model contains two parameters to be estimated: δ_1 and δ_2. The sign and the magnitude of these two parameters determine the characteristics of the function $\beta(t) = \left(1 + \exp\left(\delta_1 t + \delta_2 t^2\right)\right)^{-1}$ that can be increasing or decreasing, concave or con-

[28] Ahn et al. (1994) developed a generalized method of moments approach to the estimation of Lee and Schmidt model specified by the equations (43 and 47).

vex.[29] The function $\beta(t)$ varies between zero and one. The test of the null hypothesis of time-invariant technical efficiency can be performed by setting $H_0 : \delta_1 = \delta_2 = 0$. In this case, the function $\beta(t)$ has a constant value of 1/2. Battese and Coelli (1992) require only one parameter δ to be estimated. The function $\beta(t) = (\exp(-\delta(t-T)))$ can take any positive value. Given that the value of the second derivative is always positive,[30] and if $\delta > 0$, the function $\beta(t)$ decreases at an increasing rate. If $\delta < 0$, it increases at an increasing rate. The hypothesis of time-invariant technical efficiency can be tested by setting the null hypothesis $H_0 : \delta = 0$.

Kumbhakar and Hjalmarsson (1993) model the inefficiency term as

$$u_{it} = a_i + \xi_{it}, \tag{53}$$

where a_i is a producer-specific component which captures producer heterogeneity also due to omitted time-invariant variables, and ξ_{it} is a producer time-specific component which has a half-normal distribution. The estimation of this model is in two steps. In the first step, either a fixed-effects model or a random-effects model is used to estimate all the parameters of the model $y_{it} = \beta_0 + \boldsymbol{\beta}\mathbf{x}_{it} - u_{it} + v_{it}$, except those in equation (53). In the second step, distribution assumptions are imposed on ξ_{it} and v_{it}. The fixed effects $(\beta_o + a_i)$ and the parameters ξ_{it} and v_{it} are estimated by maximum likelihood, conditioned on the first step parameter estimates.

[29] The fist and the second derivatives of the function defined by equation (51) depend on the two parameters δ_1 and δ_2.

[30] The first and second derivatives of the function defined by equation (52) are respectively equal to: $\partial\beta(t)/\partial t = \exp\{-\delta(t-T)\}(-\delta)$; $\partial^2\beta(t)/\partial t^2 = \exp\{-\delta(t-T)\}\delta^2$.

6 A Model for Stochastic Technical Inefficiency Effects for Panel Data: Battese and Coelli 1995

The panel data framework allows to correct the bias of omitted unobservable variables. These variables are characteristics peculiar to each production unit. If these unobservable omitted variables are correlated with included variables, the estimated coefficients are biased.

The use of panel data techniques allows to solve many limitations of the cross-country method. Durlauf and Johnson (1995) postulate that cross-country differences are not explained entirely by differences in rates of physical and human capital accumulation and population growth. Initial conditions determine aggregate production opportunities that differ considerably across production units. Islam (1995) observes that the cross-country regression approach includes several explanatory variables to account for the differences in preferences and technology, and therefore in steady states. However, these differences are not measurable and observable. A panel data approach can overcome these problems by controlling for individual effects. McDonald and Roberts (1999) state that panel data method allows to analyse cross-section and time series variation in the data and to test the validity of the assumption regarding common technology implied by the cross section studies.

The inefficiency models exposed so far have not explicitly formulated a model for technical inefficiency effects in terms of appropriate explanatory variables. Battese and Coelli (1995) propose a model for stochastic technical inefficiency effects for panel data which includes explanatory variables. The panel framework permits to exploit the time and sectional dimensions of the data. The stochastic nature of the inefficiency terms, allows the estimation of both technical change - captured by time dummies - in the stochastic frontier and time-varying technical inefficiency.

Assume the following common production frontier for the production units under analysis:

$$Y_{it} = f(\mathbf{X}_{it})\tau_{it}\xi_{it} \quad i = 1, ...N; \quad t = 1, ...T \tag{54}$$

where Y_{it} is real output for country i at time t and $\mathbf{X}_{it}$ are production inputs and other factors associated with country i at time t. τ_{it} is the efficiency measure, with $0 < \tau_{it} < 1$,[31] and ξ_{it} captures the stochastic nature of the frontier. Writing a production function of the Cobb-Douglas type in log-linear form, we obtain

$$y_{it} = \mathbf{x}'_{it}\boldsymbol{\beta} + v_{it} - u_{it} \tag{55}$$

where $u_{it} = -ln\tau_{it}$ is a non-negative random variable. The composite error is $v_{it} = ln\xi_{it}$, where v_{it} is normally distributed with mean 0 and variance σ_v^2.

In matrix form, we obtain the basic panel data stochastic frontier model:

$$y_t = \mathbf{I}_{\mathrm{N}}\alpha + \mathbf{x}_t\boldsymbol{\beta} + \mathbf{v}_{\mathrm{t}} - \mathbf{u}_{\mathrm{t}} \qquad t = 1, \ldots, T, \tag{56}$$

[31] When $\tau_{it} = 1$ there is full efficiency, in this case the country i produces on the efficiency frontier.

with

$$\mathbf{y}_t = \begin{pmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ v_{N,t} \end{pmatrix}; \mathbf{x}_t = \begin{pmatrix} x_{1,1,t} & x_{1,2,t} & \dots & x_{1,k,t} \\ x_{2,1,t} & x_{2,2,t} & \dots & x_{2,k,t} \\ \vdots & \vdots & & \vdots \\ x_{N,1,t} & x_{N,2,t} & \dots & x_{N,k,t} \end{pmatrix};$$

$$\mathbf{v}_t = \begin{pmatrix} v_{1,t} \\ v_{2,t} \\ \vdots \\ v_{N,t} \end{pmatrix}; \mathbf{u}_t = \begin{pmatrix} u_{1,t} \\ u_{2,t} \\ \vdots \\ u_{N,t} \end{pmatrix}.$$

In logarithmic specification, technical efficiency of country i is defined as

$$\tau_{it} = e^{-u_{it}} \tag{57}$$

Efficiency is ranked as $u_{N,t} \leq \ldots \leq u_{2,t} \leq u_{1,t}$: Country N produces with maximum efficiency in the sample.

Often studies estimate the stochastic frontier and calculate the efficiency term, and, as a second step, they regress predicted efficiency on specific variables to study the factors which determine efficiency. But such a two-stage procedure is logically flawed.[32] It requires a first-stage assumption that the inefficiencies are independent and identically distributed. Kumbhakar et al. (1991) and Reifschneider and Stevenson (1991) address this issue by proposing a single-stage Maximum Likelihood procedure. Battese and Coelli (1995) propose an extended version of the model of Kumbhakar et al. (1991) to allow the use of panel data.[33] Battese and Coelli (1995) specify inefficiency

[32] On this argument, see Wang and Schmidt (2002).
[33] See also Koop et al. (2000b).

as

$$u_{it} = \boldsymbol{\delta}\mathbf{z}_{it} + \omega_{it}, \tag{58}$$

where u_{it} are technical inefficiency effects in the stochastic frontier model that are assumed to be independently but not identically distributed, $\mathbf{z}_{it}$ is vector of variables which influence efficiencies, and $\boldsymbol{\delta}$ is the vector of coefficients to be estimated. ω_{it} is a random variable distributed as a truncated nomal distribution with zero mean and variance σ_u^2. The requirement that $u_{it} \geq 0$ is ensured by truncating ω_{it} from below such that $\omega_{it} \geq -\boldsymbol{\delta}\mathbf{z}_{it}$. Battese and Coelli (1995) underline that the assumptions on the error component ω_{it} are consistent with the assumption of the inefficiency terms being distributed as truncated normal distribution $N^{+}(\boldsymbol{\delta}\mathbf{z}_{it}, \sigma_u^2)$.

Maximum likelihood estimation is used to take into consideration the asymmetric distribution of the inefficiency term. Greene (1980a, 1990) argues that the only distribution which provides a maximum likelihood estimator with all desirable properties is the Gamma distribution. However, following van den Broeck et al. (1994), the truncated distribution function is preferred, which better distinguishes between statistical noise and inefficiency terms.

Technical efficiency of country i at time t is

$$TE_{it} = exp\left(-u_{it}\right) = \exp\left(-\boldsymbol{\delta}\mathbf{z}_{it} - \omega_{it}\right) \tag{59}$$

Jondrow et al. (1982) suggest a measure of efficiency based on the distribution of inefficiency conditional to the composite error term, $u_{it} \mid \varepsilon_{it}$ (where $\varepsilon_{it} = v_{it} - u_{it}$). The distribution contains all the information that ε_{it} yields about u_{it}. The expected value of the distribution can therefore be used as a point estimate of u_{it}. When the distribution of the inefficiency component is a truncated distribution, a point estimate for technical efficiency TE_{it} is given by[34]

$$E\left(TE_{it}\right) = E\left[\exp(-u_{it})\,|\varepsilon_{it}\right] = \\ = \frac{\left[\Phi(-\sigma_* + \mu^*_{it}/\sigma_*)\right]}{\left[\Phi(\mu^*_{it}/\sigma_*)\right]} \exp\left[-\mu^*_{it} + \frac{1}{2}\sigma^2_*\right] \tag{60}$$

$\mu^*_{it} = \left(\sigma^2_v \boldsymbol{\delta}\mathbf{z}_{it} - \sigma^2_u \varepsilon_{it}\right)\left(\sigma^2_u + \sigma^2_v\right)^{-1}$ and $\sigma^2_* = \sigma^2_u \sigma^2_v (\sigma^2_u + \sigma^2_v)^{-1}$.[35] $\Phi(.)$ is the standard normal cumulative density function. Implementing this procedure requires estimates of μ^*_{it} and σ^2_*. In other words, we need estimates of the variances of the inefficiency and random components and of the residuals $\hat{\varepsilon}_{it} = y_{it} - \hat{\alpha} - \mathbf{x}_{it}\hat{\boldsymbol{\beta}}$.

By replacing the unknown parameters in equation (60) with the maximum likelihood estimates an operational predictor for the technical efficiency of the country i in the time period t is obtained. As opposed to the models in the previous section, these technical efficiency measures include the influence of explanatory factors. The inefficiency model in equation (58) include a shift parameter δ_0 which is constant across production units. The model treats multiple observations of the same unit as being obtained from independent samples. Therefore the model is a pooled estimator.[36]

To better exploit the data's panel nature, Kumbhakar and Hjalmarsson (1995) and Wang (2003) suggest to incorporate individual specific effects in the inefficiency model (equation 58). This extension would permit to obtain a within estimator. The truncated distribution of the inefficiency does not allow to take first differences or subtract means from the data to eliminate

[34] See Kumbhakar and Lovell (2000, p.271) and Battese and Coelli (1995). Equation (60) is similar to the cross-section version of equation (32).

[35] The following assumptions must hold : (i) the v_{it} are iid $N(0, \sigma^2_v)$, (ii) $\mathbf{x}_{it}$ and v_{it} are independent, (iii) u_{it} is independent of x and v, and (iv) u_{it} follows a one-sided normal distribution (e.g. truncated or half-normal).

[36] Battese and Coelli (1995) underline that the inclusion of the intercept parameter δ_0 is essential to have parameter estimates associated with explanatory variables $\mathbf{z}$ unbiased.

these specific effects, given that differenced truncated normal distributions do not result in a known distribution (Wang 2003). Kumbhakar (1991) suggests to include dummies to take into account specific characteristics.

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